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BY

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Second Edition, Revised and Enlarged

NEW YORK

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1909

PREFACE TO SECOND EDITION.

In this work, which has been almost entirely rewritten, elastic arches of variable cross-section, particularly concrete and reinforced-concrete arches, have been treated in great detail; the aim being not only to present the theory in a simple, convincing manner, but to furnish the computer with several examples worked in full, so that he should find no difficulty in applying the theory, rapidly and with certainty, to similar arches.

In Chapter I the elastic theory is developed. In Chapter II the application to a concrete arch is given in full. The graphical treatment is shorter than that given in the author's "Steel-concrete Arches," and the "Résumé of Operations" given should put the draftsman in complete possession of the method. The algebraic solution that follows is especially recommended, however, as it enables the pressure line to be established to any desirable degree of accuracy.

In Chapter III an algebraic solution for a three-centered arch is given in great detail for a moving load in various positions, corresponding to maximum moments at critical sections. In Chapter IV temperature and allied stresses are found, and in Chapter V the method of single loads is developed and applied to a reinforced-concrete arch. The method being simple and easily followed, the hope is expressed that this method of single loads may come into more favor than it has in the past. The book concludes, in Chapter VI, with a treatment of arches with two and three hinges.

Assistance has been derived from Professor Eddy's "Researches in Graphical Statics," particularly in the conception of placing the trial polygon in true position on the arch with respect to the line kk_1; otherwise the demonstrations and treatment throughout are original.

WM. CAIN.

CHAPEL HILL, N. C.,
Oct. 1, 1908.

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THEORY OF SOLID AND BRACED ELASTIC ARCHES.

CHAPTER I.

EQUILIBRIUM POLYGONS. ELASTIC THEORY.

1. The term *solid arch* is applied, in what follows, to arches of concrete or reinforced concrete, or to metal arches having a continuous web connecting the flanges; the term *braced arch*, to such as are braced between the flanges by the usual struts and ties, forming any pattern of open web. As contradistinguished from the voussoir arch, the solid or braced arch is capable of supplying tensile resistances, at any ideal section, when needed; though as all arches are composed of elastic materials, the term *elastic arch* is applicable to any one of them. When the solid arch is hinged at one or more points (not exceeding three), there are, of course, no tensile forces exerted at

the hinged joints, so that the hinged joints, if any, must be excepted in the above definition.

The theory of the stone or voussoir arch will fall under that pertaining to the solid arch when only compressive forces are exerted on the actual joints, as is illustrated in the author's "Theory of Voussoir Arches."

Before entering upon the theory of the elastic arch, it is well perhaps to make some remarks on equilibrium polygons, particularly those pertaining to the barrel or cylindrical arch, the only kind treated in this book.

THE USUAL METHOD OF DRAWING AN EQUILIBRIUM POLYGON.

2. The subject is easily introduced by considering a portion of a concrete arch contained between two vertical planes, one foot apart, both planes being perpendicular to the axis of the arch. Fig. 1 is supposed to represent a mid-section of such a lamina of a semi-arch, and it will be assumed that the thrust S at the crown acts at *a* and is known likewise in magnitude and direction. The weights of the arch and load contained between the consecutive verticals at *a*, *b*, *c*, and *d* can be easily computed and will be designated

P_1, P_2, P_3, respectively. They, of course, act vertically through the centers of gravity of the corresponding trapezoids of the mid-section. To the left of the arch is drawn the *force diagram* as follows: From some convenient point O, lay off S in direction and magnitude to the scale of

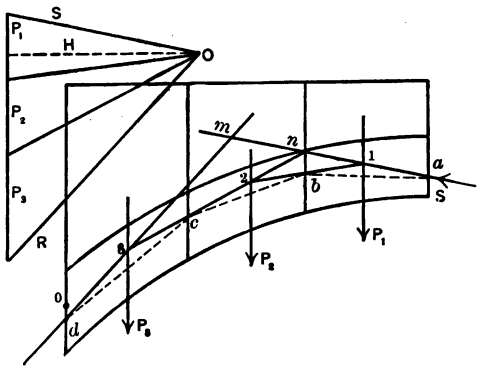

Fig. 1.

loads; from the left extremity, lay off, to the same scale of loads, successively P_1, P_2, P_3 vertically downwards, and from the points of division draw *rays* to O, the *pole*. Any ray will be designated by the letters either side of it.

The force diagram gives the resultants on the sections at *a*, *b*, *c*, *d* in magnitude and direction only. Thus the resultant in

magnitude and direction of S and P_1 is ray P_1P_2; of ray P_1P_2 and P_2 is ray P_2P_3; of ray P_2P_3 and P_3 is ray P_3R. The lines of action of these resultants on the arch are found as follows: Draw S1 ∥ ray SP_1 to intersection 1 with P_1; then since the resultant of S and P_1 acts parallel to ray P_1P_2, draw 12 ∥ ray P_1P_2, to give its line of action on the arch. This intersects P_2 at 2. Similarly, since the resultant of ray P_1P_2 and P_2 is ray P_2P_3, the line of action on the arch of the resultant P_2P_3 is 23 ∥ ray P_2P_3 It intersects P_3 at 3, where $3d$ ∥ ray P_3R gives the line of action of the resultant of all the forces acting to the *right* of *d*.

The polygon $a123d$ is called the *equilibrium polygon*, or *line of pressures*, since the resultants, which in magnitude equal rays SP_1, P_1P_2, P_2P_3, P_3R, act respectively along $a1$, 12, 23, $3d$.

These resultants meet the vertical sections at *a*, *b*, *c*, *d*, and the dotted line *abcd* is called the *line of resistance*, or line of the centers of pressure.

Now produce 32 to intersection with S at *n*; then *n* is a point on the resultant of P_1 and P_2 in position on the arch, since on combining this resultant of P_1 and P_2 with S acting along *an*, the final resultant must coincide with $n23$ as found above. Similarly, the resultant of P_1, P_2, and P_3

must act through *m*, the intersection of *d*3 produced with *am*, since 3*d* is the line of action of the resultant of S, P_1, P_2, P_3, as found above.

Where a large number of forces P_1, P_2, . . . are used, mistakes as to drawing a side of the equilibrium polygon parallel to the wrong ray can be entirely avoided by use of the notation above: 12 ∥ ray P_1P_2, 23 ∥ ray P_2P_3, etc., the subscripts having the same numbers as the sides.

3. *Moments.*—To find the moment about *o*, the center of section at *d*, of the forces S, P_1, P_2, P_3 acting on the arch from the crown to the section; since the resultant of these forces is equal in magnitude to ray P_3R, to the scale of loads, and it acts along 3*d*, the moment required is ray P_3R multiplied by the length of the perpendicular, to the scale of distance, let fall from *o* upon 3*d*. Another expression for this moment is found by decomposing the resultant (= ray P_3R in magnitude) at *d* into a horizontal component H and a vertical component, and taking moments about *o* of the components. Since the vertical component acts in the line *od*, its moment about *o* is zero; hence the moment of the resultant is simply H . *od* and is negative if we regard left-handed couples as having positive moments. If

$3d$ passed above o, the moment would be positive. The above principle applies to any section.

On dropping from pole O a perpendicular upon the load line, its length is called the *pole distance*, and to the scale of loads its value equals H, the horizontal component of *any* of the resultants S, ray P_1P_2, etc. Thus where the loads are vertical, the thrusts at any sections have the same horizontal component H. If the thrusts at sections a, b, c, acting along the lines $a1$, 12, 23, respectively, cut the sections at points distant t_a, t_b, t_c from their centers, the moments about such centers of all forces to their right are, respectively, Ht_a, Ht_b, Ht_c, these moments being positive when the thrusts pass above the centers of the sections, negative otherwise.

4. *Computation of Ordinates of Equilibrium Polygon.*—In the trial equilibrium polygons to be subsequently drawn, the thrust H at the crown is assumed horizontal. In Fig. 2 the loads P_1, P_2, . . . , P_r are distant x_1, x_2, . . . , a from P_s at e. The equilibrium polygon $1234n$ is drawn as detailed above, 12 $\parallel$ ray P_1P_2, 23 $\parallel$ ray P_2P_3, etc., and the sides 12, 23, . . . are produced to meet the vertical at e in f, g, ...

From the similarity of triangles in the

upper figure and the force diagram, we have

$$\frac{P_1}{H}=\frac{ef}{x_1},\quad \frac{P_2}{H}=\frac{fg}{x_2},\quad \ldots,\quad \frac{P_r}{H}=\frac{hn}{a}.$$

On clearing of fractions and adding,

$$m_s=P_1x_1+P_2x_2+\ \ldots\ +P_ra=H\,.\,en.$$

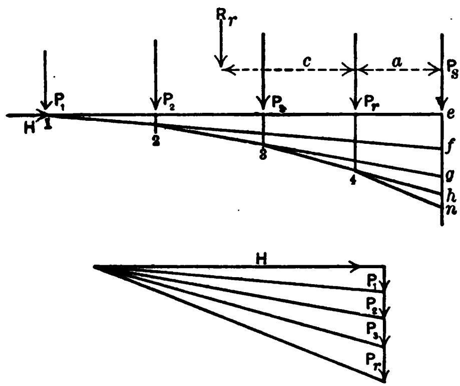

FIG. 2.

The left member is the sum of the moments of the loads about e or P_s. Suppose this computed and that H is known, then $en=m_s\div H$ can be computed. Similarly for the ordinates at 2, 3, ... In this way the polygon 123 ... n can be drawn with great accuracy. The com-

putation of m_s is facilitated by a formula to be derived. Here m_s is the moment of all loads from the crown up to P_s about P_s. Similarly, let m_r be the moment of all loads from the crown up to P_r about P_r. Also call the resultant or sum of the loads from the crown up to and including P_r, R_r, and suppose this resultant acts c feet from P_r; then $m_r = R_r c$ and $m_s = R_r(c+a)$.

$$\therefore m_s = m_r + R_r a. \quad . \quad . \quad . \quad (1)$$

It will be seen in subsequent articles how much this simple formula facilitates the computation of m_s.

The formulas above are all true when the thrust S at the crown is not horizontal, but acts along a line inclined to the horizontal. In this case, however, H is the horizontal component of S, or the common altitude of all the triangles in the force diagram.

SOME PROPERTIES OF THE EQUILIBRIUM POLYGON WITH END MOMENTS.

5. In Fig. 3 suppose a_1aa_2 to be the neutral line of an arch * subjected to a single force P. As we are not yet prepared to draw the true equilibrium polygon, we draw a trial one to investigate

* Defined in Art. 8.

some of its properties to be utilized hereafter.

Lay off P to scale of force equal to the vertical line of the force diagram; assume any pole O and draw R_1 and R_2 from O to the extremities of P. Then from any point c_3 on P, draw $c_1c_3 \parallel R_1$, $c_3c_2 \parallel R_2$ to intersections c_1 and c_2 with the verticals through a_1 and a_2. The reaction R_1 at c_1, acting from c_1 towards c_3, and the reaction R_2 at c_2, acting from c_2 towards c_3, will hold P at c_3 in equilibrium. From O draw F arbitrarily to cut the load line P into the two components V_1 and V_2. Then the reactions R_1 and R_2 at c_1 and c_2 can be supposed decomposed into the components V_1 and F at c_1, V_2 and F at c_2, acting as indicated by the arrows. Thus P at c_3, V_1 and F at c_1, and V_2 and F at c_2 form a system in equilibrium.

Next, draw an arbitrary line k_1k_2 parallel to F, and suppose applied two opposed forces F at k_1 and two opposed forces F at k_2, all acting along the line k_1k_2. This does not destroy equilibrium, but since the two forces F acting towards each other balance, the effect is to form a couple at a_1 with F at c_1 and the remaining F at k_1; also a couple at a_2 with F at c_2 and the remaining F at k_2. If H is the horizontal component of the equal forces F, on decomposing them at c_1, k_1,

c_2, k_2 into vertical and horizontal components, we note that, since the vertical components act along k_1c_1 and k_2c_2 respectively, the moment of the couple at a_1 is $H \cdot k_1c_1$, that at a_2 is $H \cdot k_2c_2$. We shall

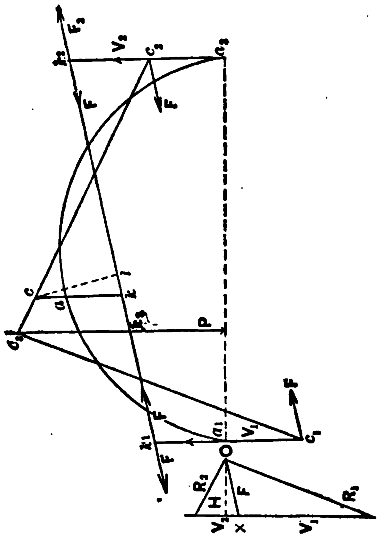

FIG. 3.

assume a fictitious structure subjected to these end couples, also to V_1 at a_1, V_2 at a_2, and P at c_3. As we have seen, the system is in equilibrium.

Take any point c in either side of the

equilibrium polygon and designate by x and d the arms of V_1 and P about c. The moment of the external forces to the left of c about c is

$$M = (V_1 x - H \cdot k_1 c_1 - Pd),$$

if c is to the right of P. If c is to the left of P, the term Pd is omitted.

Another simpler expression can be found for M. Thus supply again the two forces F acting towards each other at k_1 and k_2; also suppose two opposed forces R_1 acting along c_1c_3, the one acting down at c_1, the other up at c_3, and two opposed forces R_2 acting along c_3c_2, the one acting up at c_3, the other down at c_2; then we see, by reference to the force diagram, that there is complete equilibrium at each vertex, k_1, c_1, c_3, c_2, and k_2. Hence the sum of the moments of the supposed forces acting at k_1, c_1, and c_3 about c will be zero. Omit now the opposed forces R_1 acting along c_1c_2.

$$\therefore\ (V_1 x - H \cdot k_1 c_1 - Pd) - F \cdot cl = 0.$$

cl is here drawn perpendicular to k_1k_2. On drawing ck vertical and resolving F acting to the right at k into a horizontal component H and a vertical component, the moment of the latter about c is zero;

hence the moment F . *cl* of F about *c* can be replaced by H . *kc*.

$$\therefore\ M = H \,.\, kc.$$

If *c* is taken below the *closing line* k_1k_2, as we shall call it, the moment of F, and therefore M, changes sign. In words this formula states that *the moment* M (as defined above) *about any point in the vertical kc is equal to the pole distance* H, *to the scale of force, multiplied by the ordinate kc from the closing line to a side of the equilibrium polygon, measured to the scale of distance.*

If there are a number of forces P acting vertically, the equilibrium polygon *c* is formed as usual, and it is easily seen that the above relation holds for any point *c*.

Should it be desired to draw k_1k_2 horizontal, not changing H, V_1, V_2, P, or the end couples, then $M = V_1x - H \,.\, k_1c_1 - Pd$ is unchanged; $\therefore kc = M \div H$ is unchanged, so that we simply draw k_1k_2 horizontal and lay off at k_1, k_3, and k_2 the ordinates k_1c_1, k_3c_3, and k_2c_2, the same as before. Otherwise, in the force diagram, at the point where F intersects the load line, draw a horizontal to the right equal to the old pole distance H to fix O′, the new pole. With this new force diagram the new

polygon is drawn as before. On using the former values of k_1c_1, k_2c_2, it will be found that k_1k_2 is horizontal, and kc for any x will be the same as before. Let the student make the construction.

From the relation $M = H \,.\, kc$, it is seen that if kc is altered in a given ratio, then H will be altered in the inverse ratio.

In the constructions required for the elastic arch, the trial equilibrium polygon b (see Fig. 11) is first drawn and its closing line mm_1 is then drawn to satisfy certain conditions; then as above, mm_1 is supposed to take a certain horizontal position kk_1 on the arch, and finally the ordinates from mm_1, now kk_1, are altered in a certain ratio and the pole distance in the inverse ratio. In all of these changes it is seen that V_1 and V_2 are unchanged, the line from O cutting off these segments being drawn parallel to k_1k_2 of Fig. 3, or to mm_1 of Fig. 11.

It will eventually be shown that the trial polygon b, after the transformations, has become the true polygon c on the arch, the end moments of the arch being $H \,.\, a_1c_1$, $H \,.\, a_2c_2$, the pole distance H of polygon c being the horizontal thrust of the arch.

THE REINFORCED ARCH.

6. Formulas for unit stresses, deflections, etc., will now be derived for the reinforced arch—the most general case; the results for arches of steel, stone, or concrete will follow at once as special cases. Let us consider a concrete arch with steel

bars embedded in the concrete, as shown by the dotted lines in the longitudinal section, Fig. 4, and in cross-section by the little rectangles in Fig. 5, representing a cross-section of the arch anywhere. The pair of bars in the same vertical plane will be called a *rib*, and they may be of any pattern (angles, plates, etc.), and connected by latticing if preferred, though the

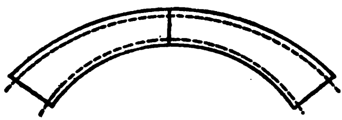

FIG. 4.

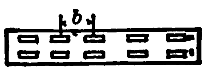

FIG. 5.

latter may possibly be omitted, since the concrete is generally capable of taking the shearing forces (exerted at right angles to the ribs), which are small. It is understood that the steel bars are free of paint, oil, scale, or rust, so that when embedded the adhesion between the steel and concrete will be complete and sufficiently great to cause the concrete and steel to

act as one mass, or, preferably, a mechanical bond between the steel and concrete is used.

7. The steel ribs are generally spaced uniformly a few feet apart, and in consequence a very rough approximation has to be resorted to in practice to apply theory to the really very complicated case.

It is assumed that it is approximately correct to consider the material of the upper bars to be distributed uniformly along an arch sheet that passes through the center lines of the upper bars, and that the material of the lower bars is similarly distributed uniformly along an arch sheet that passes through the center lines of the lower bars; so that if A = area of cross-section of the two bars constituting a rib, in square feet, and if the ribs are spaced b feet apart, then $A \div b = A_2$ will be the area in square feet of the cross-section of steel supposed in a slice of the arch contained between two vertical longitudinal planes one foot apart. It would be safer, perhaps, to allow only a fraction of A_2 in the computation, and certainly it would seem advisable to limit b to a certain maximum, but as this must remain for the present a matter of judgment, the simple assumption above will be made in what follows, and A_2 in the formulas can be altered to suit the judgment of the

engineer. Therefore a longitudinal slice of the arch contained between two vertical planes one foot apart will be assumed to have the cross-section, Fig. 6.

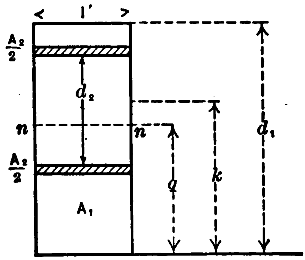

FIG. 6.

In this cross-section, which is properly taken perpendicular to the neutral surface and approximately at right angles to the soffit,

A_1 = area of concrete in square feet;
A_2 = area (shaded) of two steel bars in square feet $= A \div b$;
d_1 = depth of arch in feet;
d_2 = depth of steel rib in feet;
k = distance in feet from soffit to center of gravity of steel rib.

The modulus of elasticity in pounds per square foot,

$$\text{for concrete} = E_1;$$
$$\text{for steel} = nE_1 = E_2.$$

8. *Stresses Corresponding to Uniform Shortening. Position of Resultant. Gravity Axis.*—The fundamental formula connecting stress and deformation is

$$f=\frac{\text{Elongation of fiber}}{\text{Length of fiber}}\times \mathrm{E}, \quad . \quad (1)$$

where f = stress in pounds per square *foot* if E is expressed in pounds per square *foot*. Suppose a normal stress to be applied to the section that shortens all the fibers an equal amount. This entails a uniform compressive unit stress, say p on the concrete, acting on the area A_1, and by (1) a unit stress np on the steel acting on an area A_2, since we have assumed $E_2=nE_1$. The resultant $p(A_1+nA_2)$ acts at a distance q' above the lower edge of the section, and taking moments about that edge,

$$q'p(A_1+nA_2)=p(A_1\frac{d_1}{2}+nA_2k).$$

Now if for the given steel section we substitute n times its given area at the same distances from the edges, the area of the "revised section," as we shall call it, is A_1+nA_2. But to find the distance q from the lower edge of the revised sec-

tion to its center of gravity, we have, on taking moments about the lower edge,

$$q(A_1 + nA_2) = (A_1 \frac{d_1}{2} + nA_2k),$$

which determines q and the gravity axis nn_1. As this formula is identical with the preceding, on dividing by p, it is seen that $q = q'$.

Thus a uniform shortening of the fibers at the cross-section entails that the resultant of the stresses shall act through the center of gravity of the "revised section," and, *conversely*, if the normal force on the section acts through the center of gravity of the revised section, it will cause a uniform shortening of the fibers there.

9. In the next figure (7), the *neutral surface nn'* is defined to be that surface where the stress due to bending moments *only* is zero. It will be shown in Art. 11 that this surface practically coincides with the gravity axis at each section for usual arch rings.

Fig. 7 gives a side view of a part of the supposed arch, 1 foot wide, contained between two planes perpendicular to the neutral surface nn' and making an angle α in circular measure before strain between them. A vertical plane midway between the faces of the supposed arch intersects

the neutral surface in the line $nn' = \Delta s$ feet in length, which may be called the *neutral line*, and the forces acting upon the artificial voussoir considered will have their resultants acting in this medial plane; hence the problem is referred to one of forces acting in one plane.

Let R be the resultant of all external forces acting upon the section passing

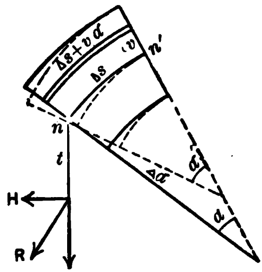

FIG. 7.

through n, the forces considered being the right reaction, and all loads acting on the arch from the right abutment up to the section (or joint) passing through n. As we have seen, when the true equilibrium polygon has been located, the line of action of R is given by the side of the equilibrium polygon pertaining to n, and its amount and direction, from the ray of the force diagram, parallel to this side.

As α and Δs will be considered very

small, the voussoir can be considered without weight or loads acting on it, and be treated as a free body acted on by R and the reaction stresses of the part of the arch to the left of n upon the section through n. To ascertain these stresses, conceive applied at n two opposed forces $+$ R, $-$ R, each equal and parallel to R. The single force R is thus replaced by a couple $R\overline{R}$ and a force $+$ R acting at n. The latter may be decomposed into components T and N tangential and normal to nn' at n. The force T will be considered in Art. 15. The force N acting along the section is the shearing force, and having but a small effect in the deformation of the arch, is neglected.

10. The couple $R\overline{R}$ is principally effective in changing the curvature of the arch, and its moment is most conveniently found by multiplying the horizontal component of R = H, the pole distance, by the vertical distance from n to R. Thus call this distance in feet $= t$; then if R is resolved, where the vertical through n cuts it, into a horizontal component H and a vertical component, the latter acts through n; hence the moment M of R about n = moment of couple $R\overline{R} = Ht$.

$$\therefore\ M = Ht \text{ (\textit{in foot-pounds})},$$

when H is in pounds and t in feet.

Let us regard t as positive when R passes above n, and negative when R passes below n. M will take the sign of t.

Under the action of this couple, the angle α is changed to α', and the curvature is increased if R cuts the section below n (as then the greatest compression is at the intrados), and decreased when R cuts the section above n. If we call $\Delta\alpha = \alpha' - \alpha$, then $\Delta\alpha$ is positive when M is negative, and therefore R acts below n; $\Delta\alpha$ is negative when M is positive, and consequently R acts above n.

11. Call the distance of any fiber from $nn' = v$, this being positive for a fiber above nn', negative below. As nn' is very small, it can be treated as an arc of a circle, and the axis of a fiber in the same plane as concentric with it. The length of the fiber before flexure is $(\Delta s + v\alpha)$, after flexure, $(\Delta s + v\alpha')$; its change of length is $v(\alpha' - \alpha) = v \, . \, \Delta\alpha$. Calling its cross-section $= a$ in square feet, and the unit stress on it due to M $= f$ in pounds per square foot, the stress on the fiber, if of concrete, is

$$fa = \frac{v \, . \, \Delta\alpha}{\Delta s + v\alpha} a\mathrm{E}_1 , \quad . \quad . \quad . \quad (2)$$

and if of steel,

$$fa = \frac{v \, . \, \Delta\alpha}{\Delta s + v\alpha} an\mathrm{E}_1 \quad . \quad . \quad . \quad (3)$$

by formula (1) above.

It is plain that $(\Delta s + v\alpha)$ in the denominators can be replaced by Δs, without appreciable error, when the radius of the intrados is large compared with d_1. The sum of all the stresses (due to flexure only) acting on the entire section at n is, therefore,

$$\Sigma(fa) = \frac{E_1 \Delta\alpha}{\Delta s} \Sigma(va), \quad . \quad . \quad . \quad (4)$$

where a is to be replaced by na in the summation for the steel bars, which is the same as if at the distance of each steel bar from the neutral axis, n times the same area of concrete was taken.

Recurring to the free body, Fig. 7, in equilibrium under the action of R and the resisting stresses along section n, since the algebraic sum of the components of all these forces perpendicular to section at n equals zero (by a law of mechanics), and since T, the component of R, is directly balanced by the stresses that cause a uniform shortening of the fibers on the section (Art. 8), it follows that the remaining normal stresses (due to M) must balance independently:

$$\therefore \ \Sigma fa = 0 \quad \text{or} \quad \Sigma va = 0,$$

which shows that the neutral axis nn' passes through the center of gravity of

the revised area of the section, and hence is at a distance q, as given in Art. 8, from the soffit.

12. *Change in the Inclination of End Tangents.*—The moment of the stress (af) on any fiber about n is (afv):

$$\therefore \text{ M} = \Sigma(afv) = \text{E}_1\frac{\Delta\alpha}{\Delta s}\Sigma(v^2a)$$

for the revised area.

Designating by I_1 the moment of inertia of the concrete (of area A_1), and by I_2 the moment of inertia of the actual area of steel, A_2 (Fig. 6), both in feet,

$$\Sigma v^2a = \Sigma(v^2a) \text{ for concrete} + \Sigma(v^2an) \text{ for steel} = \text{I}_1 + n\text{I}_2.$$

$$\therefore \quad \text{M} = \text{E}_1\frac{\Delta\alpha}{\Delta s}(\text{I}_1 + n\text{I}_2).$$

$$\therefore \ \Delta\alpha = \frac{\text{M}\Delta s}{\text{E}_1(\text{I}_1 + n\text{I}_2)}. \quad . \quad . \quad (5)$$

By reference to Fig. 7, and noting that the sections were drawn perpendicular to the neutral axis, it is seen that (5) gives $\Delta\alpha$, or *the change* in the angle between the tangents to the neutral line at n and n', due to the couple $\text{R}\overline{\text{R}}$, whose moment is M. This approaches the exact truth as near as we please, as α and Δs

approach zero indefinitely. Therefore, if we could proceed by analysis alone, $\Delta\alpha$ and Δs would be replaced by $d\alpha$ and ds in (5), and the result integrated to find the change in the inclination of the end tangents of the neutral line, corresponding to a length s measured along that arc.

13. To apply the graphical method, however, an approximation must be introduced here whose significance must be carefully noted. The assumption is that for an appreciable length of Δs (several feet, for instance), $\Delta\alpha$ is given by (5), provided M is taken as constant and equal to the value corresponding to the mid-point of nn', Fig. 7, or $\frac{1}{2}\Delta s$ distant from either n or n', E_1, I_1, and I_2 being likewise taken there.

As the total change in the inclination of the end tangents for a length s is the sum of all the infinitesimal changes for the part of the arch considered, or

$$\Sigma\left(\frac{\Delta s}{E_1(I_1+nI_2)}M\right),$$

Δs being very small, the assumption above is that this expression is equal approximately to

$$\theta=\frac{s}{E_1(I_1+nI_2)}M_0,$$

where $s=\Sigma(\Delta s)$, M_0 is the moment at the middle of s, and E_1, I_1, and I_2 the corresponding quantities at the same point.

This assumption, though not exact, is the most reasonable that can be made, but it can only be tested for I_1 and I_2 variable in a numerical example. If E_1, I_1, I_2 are constant, as for an arch ring of the same cross-section, and E_1 constant throughout, and Δs is also constant, then the previous supposed equality reduces to

$$\Sigma(M \,.\, \Delta s) = M_0 s;$$

so that if s is laid off along a line and divided into the equal lengths Δs, and ordinates M are laid off (say) at the middle of each Δs, then $\Sigma(M\Delta s)$ represents an area and M_0 is its mean ordinate. Now, as M is proportional to t (§ 5), and calling t_0 the value of t corresponding to M_0, if the true equilibrium curve for the arch pertaining to the space s considered satisfies the condition $\Sigma(t\Delta s) = t_0 s$, then the assumption is exactly realized. This will be nearly true if the successive values of t are equal or are increasing in going from one end of s to the other, or decreasing over the same length, but not for the case where t increases over a part of s and decreases over the remainder, as a rule. The extreme limit of error is reached when t_0, the vertical distance from the middle of s on the neutral axis to the equilibrium curve, is greater than any of the other t's over the same length s. This cannot be guarded against in advance, but a study of equilibrium curves shows that it can generally happen on only two divisions of the neutral line, and even here the error is often slight in consequence of the curve running nearly parallel to the neutral line for a good part of a usual division.

14. *Unit Stresses due to Bending.*—Let f_1 = stress per square foot on an extreme fiber of the concrete, whose distance from

the neutral axis is v_1 feet; then from (2), $f_1 = v_1 E_1 \frac{\Delta\alpha}{\Delta s}$, and eliminating $\frac{\Delta\alpha}{\Delta s}$ between this equation and (5), we find

$$f_1 = \frac{Mv_1}{I_1 + nI_2}.$$

Similarly, the stress per square foot in an extreme fiber of the steel $= f_2$, distant v_2 feet from the neutral axis, from (3) and (5) is

$$f_2 = \frac{Mv_2}{I_1 + nI_2}.$$

These two stresses are due entirely to the couple whose moment is M.

15. *The stresses due to* T, the component of R acting normal to the section at n, have now to be evaluated. As shown in Art. 8, the force T causes a uniform shortening of the fibers. This uniform shortening entails a uniform compressive unit stress $= p$ on the concrete acting on the area A_1, and a unit stress np on the steel (both in pounds per square foot) acting on an area A_2. Then, as in Art. 8,

$$T = p(A_1 + nA_2),$$

from which $p = T \div (A_1 + nA_2)$,

and $$np = nT \div (A_1 + nA_2).$$

16. *Total Unit Stresses.*—From the last two articles are derived the total unit stress s_1 in pounds per square foot, exerted on the concrete at the upper or lower edges of the cross-section,

$$s_1=\frac{T}{A_1+nA_2}\pm\frac{Mv_1}{I_1+nI_2}, \quad . \quad . \quad (6)$$

and the total unit stress s_2 in pounds per square foot, experienced by the upper and lower bars of the steel,

$$s_2=\left(\frac{T}{A_1+nA_2}\pm\frac{Mv_2}{I_1+nI_2}\right)n. \quad . \quad (7)$$

All dimensions in these formulas are in feet.

For sections symmetrical with respect to the neutral line, Fig. 6,

$$q=\frac{d_1}{2}, \quad v_1=\frac{d_1}{2}, \quad v_2=\frac{d_2}{2},$$

$$I_1=\frac{d_1^{\,3}}{12}, \quad I_2=\tfrac{1}{4}A_2d_2^{\,2}=A_2v_2^{\,2}.$$

Therefore

$$\left.\begin{aligned} s_1&=\frac{T}{A_1+nA_2}\pm\frac{Mv_1}{\frac{1}{12}d_1^{\,3}+nA_2v_2^{\,2}},\\ s_2&=\left(\frac{T}{A_1+nA_2}\pm\frac{Mv_2}{\frac{1}{12}d_1^{\,3}+nA_2v_2^{\,2}}\right)n. \end{aligned}\right\} \quad (8)$$

Since the slice of the arch considered is one foot thick, for a concrete or reinforced-concrete or stone arch, $A_1 = d_1 \times 1 = d_1$. Also for a concrete arch without reinforcement, the formula for s_1 gives the maximum stresses on simply making $n = 0$.

For a steel arch consisting of parallel flanges and a solid web, A_1 is the total area of cross-section and I_1 is its moment of inertia about the gravity axis. With this designation, and calling $d_1 = 2v_1$ the depth of rib, formula (6) gives the maximum stresses in the flanges on putting $n = 0$.

For the braced arch it is more convenient to use the method of Art. 87.

Note.—If the resultant force acting on a cross-section of the arch ring meets it at a distance z from its center of gravity, as found in Art. 8, we can replace M by Tz in the formulas. When the fiber stress is zero at an edge, on placing $s_1 = 0$ in (6), using the minus sign for the second term, the equation can then be solved for z, giving the distance the resultant must act from the center of gravity so as to cause no stress in the edge on the other side of the center from the resultant. Two points can be thus found, called *core* points. If the resultant acts between the two (in the core), there can be only compression at either edge; if it acts outside the line joining the two

points, tension is exerted at one edge. For a concrete arch $n=0$, and putting $s_1=0$, we find $z=\frac{1}{6}d$; so that if the resultant on the section lies in its "middle third," only compression will be exerted throughout the cross-section.

CONDITIONS FOR EQUILIBRIUM OF ARCH WITH NO HINGES.

17. The neutral line, for the case where the steel bars are symmetrically placed with respect to the center line of the arch ring, is the center line itself, and can at once be laid off.

For an unsymmetrical cross-section, points in the neutral line must be found and laid off by aid of the formula for q, Art. 8, and a curve traced through them.

In either case let *abc*, Fig. 8, represent the neutral line of an unstrained arch with fixed end tangents, and let s represent a length of the neutral line whose center is at b. When the arch is loaded either with its own weight only or, in addition, with a live load, the neutral line changes shape, and it was agreed in Art. 13 to regard the change in the inclination of the end tangents to the neutral arc s, as given by the formula

$$\theta=\frac{Ms}{E_1(I_1+nI_2)},$$

where n and E_1 are constant, and M, I_1, and I_2 are taken at b, the middle of arc s.

Regard the end c temporarily free, then the bending in s alone will cause a rotation of the arc bc about b equal to θ, so that a line bc will rotate through an infinitesimal distance ce, taken as perpendicular to bc. Take c as origin, ca the axis of x and axis of y vertical, and call the co-ordinates of b, x and y. Draw ed

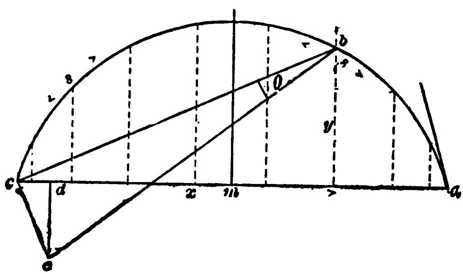

FIG. 8.

perpendicular to ca; then from similarity of triangles,

$$cd=\frac{ce}{bc}y=y\theta, \qquad de=\frac{ce}{bc}x=x\theta.$$

The exact result would of course be found by dividing s into infinitesimal lengths Δs, and regarding the rotation to take place about the end of each little

portion in turn, x and y thus having values above and below the means taken for b (the middle of s). We assume, therefore, in the graphical treatment to follow, if M, I_1, I_2, x, y are all taken at the mid-point of arc s, as a sort of average, that the horizontal and vertical deflections of c, due to s, are given nearly by the above equations.

The total horizontal and vertical displacements of c due to the bending of all portions of the arch are then given by $\Sigma(y\theta)$, $\Sigma(x\theta)$, or

$$\Sigma\frac{\mathrm{M}sy}{\mathrm{E}_1(\mathrm{I}_1 + n\mathrm{I}_2)}, \quad \Sigma\frac{\mathrm{M}sx}{\mathrm{E}_1(\mathrm{I}_1 + n\mathrm{I}_2)},$$

respectively, the summation including all the segments of the arch. The total *change* of inclination of tangents at a and c is, similarly,

$$\Sigma\theta = \Sigma\frac{\mathrm{M}s}{\mathrm{E}_1(\mathrm{I}_1 + n\mathrm{I}_2)}.$$

For an arch "fixed at the ends" and with no hinges, these three sums are zero; hence we have the three conditions to be fulfilled, corresponding to end tangents fixed in direction, span invariable, and vertical deflection of c with respect to a, zero:

$$\Sigma \frac{Ms}{E_1(I_1 + nI_2)} = 0. \quad . \quad . \quad . \qquad (9)$$

$$\Sigma \frac{Mys}{E_1(I_1 + nI_2)} = 0. \quad . \quad . \quad . \qquad (10)$$

$$\Sigma \frac{Mxs}{E_1(I_1 + nI_2)} = 0. \quad . \quad . \quad . \qquad (11)$$

All dimensions being taken in feet, and M expressed in foot-pounds.

DEFLECTION AT THE CROWN.

18. When by methods to be given later the values of t have been accurately *computed* at the center of each division s, say to 0.001 ft., then the downward deflection at the crown for the loading can be closely predicted.

In the discussion of the preceding article, if c is taken at the crown and x reckoned from that point, we find $de = x\theta$, the deflection of c due to the bending of one division s, as before; hence the lowering of the crown is given by the formula

$$\Delta y = -\Sigma \frac{Mxs}{E_1(I_1 + nI_2)} = -\Sigma \frac{Htxs}{E_1(I_1 + nI_2)}, \qquad (12)$$

where the summation extends over the semi-arch only. The minus sign is used because, by the convention of Art. 10, the

deflection of c is downwards when t is negative, upwards when t is positive. Of course the signs of t must be regarded in the summation. $\dfrac{Hs}{E_1(I_1+nI_2)}$ can be put before the sign Σ if the arch ring is divided so that $s \div E_1(I_1+nI_2)$ is constant.

DIVISION OF THE NEUTRAL AXIS.

19. Recurring to the conditions (9), (10), (11), to be fulfilled by the equilibrium polygon for an arch "fixed at the ends," replacing M by Ht and regarding E_1 (the modulus for concrete) as constant throughout the arch ring, we note, if the neutral line can be so divided that $s \div (I_1 + nI_2)$ is constant for each division, that $\dfrac{H}{E_1} \dfrac{s}{(I_1+nI_2)}$ can be placed before the sign of summation and the three conditions (9), (10), (11) reduce to

$$\Sigma t=0, \quad \Sigma(tx)=0, \quad \Sigma(ty)=0. \quad . \quad (13)$$

It will be shown in detail in Arts. 20 and 63 how this division can be easily made for both concrete and reinforced-concrete arches. The method to follow for steel arches is thus plainly indicated.

CHAPTER II.

APPLICATION OF THE ELASTIC THEORY TO A PLAIN CONCRETE ARCH.

DIVISION OF THE NEUTRAL AXIS.

20. The object is to divide the neutral line from the springing to the crown into n lengths, $s_1, s_2, \ldots, s_n$, so that

$$\frac{s_1}{I_1}=\frac{s_2}{I_2}= \ldots =\frac{s_n}{I_n}=\text{constant.}$$

Here s_1 is the first length from the springing and I_1 the value of I at its mid-point; s_2 is the second length and I_2 the value of the moment of inertia I at its mid-point, and so on.

For a plain concrete arch, $I=\frac{1}{12}d^3$, where d is the radial depth of the arch at the center of s, so that the above condition may be put in the form

$$\frac{s}{d^3}=\text{constant.}$$

A practical solution can be made by the trial-and-error method, which will be illustrated by an example.

In the arch shown in Fig. 11, the radial depths d at various distances l from the center of the springing, measured along the neutral line, were measured, and the

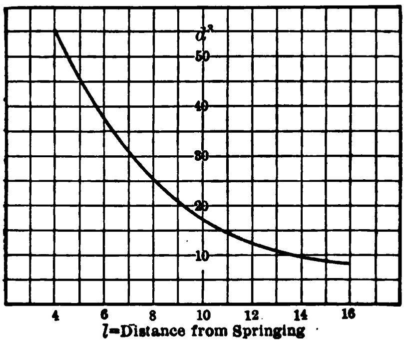

FIG. 9.

values of d^3 plotted to scale as ordinates in Fig. 9, the corresponding values of l being laid off as abscissas. If we desire to divide the semi-arch into about n divisions, calling the length of the neutral axis from the springing to the crown l_c, then l_c/n will be the average length of one division, and if d_a is the depth midway between the spring and the crown, we have, approximately,

$$\frac{l_c/n}{d_a^3} = \text{constant above.}$$

Thus by measurement in Fig. 11 we find $l_c = 17.6$ ft., $d_a = 2.78$, and if we desire about 4 divisions,

$$\frac{l_c/n}{d_a{}^3} = \frac{17.6/4}{2.78^3} = 0.205.$$

As the result is not exact, and besides, as generally, one division more or less is of no moment, the value of the ratio can be changed slightly for ease of computation. Thus write

$$\frac{s}{d^3} = 0.2; \quad \therefore\ s = 0.2d^3,$$

which will suffice for a first trial.

Let l = distance along the neutral axis from the springing to the *middle* of s_1, s_2, . . . , s_n, in turn. For any value of l, Fig. 9 gives the value corresponding of d^3, which is used below. The work now proceeds as follows:

Try $s_1 = 8$. At $l = \frac{1}{2}s_1 = 4$, $d^3 = 56$.

$\therefore\ s = 0.2d^3 = 0.2 \times 56 = 11.2$. Thus the assumed s_1 is too small.

Try $s_1 = 10$, $\therefore\ l = 5$, $s = 0.2 \times 46 = 9.2$, or the assumed s_1 is too great.

Try $s_1 = 9.6$, $\therefore\ l = 4.8$, $s = 0.2 \times 47.8 = 9.56$.

The assumed and computed s_1 now practically agree.

Next we assume $s_2 = 4$, $\therefore\ l = s_1 + \frac{1}{2}s_2 = 9.6 + 2$. $\therefore\ s = 0.2 \times 13.3 = 2.7$.

Try $s_2=2.8$, $\therefore$ $l=9.6+14=11$, $\therefore$ $d^3=14.2$, and $s=0.2\times14.2=2.8$. $\therefore$ $s_2=2.8$

Try $s_3=2$, $l=s_1+s_2+\frac{1}{2}s_3=9.6+2.8+1=13.4$, $\therefore$ $s=0.2\times10=2$. $\therefore$ $s_3=2$.

Try $s_4=1.6$, $l=(s_1+\ldots+s_3)+\frac{1}{2}s_4=15.2$, $\therefore$ $s=0.2\times8.6=1.7$. $\therefore$ $s_4=1.7$.

$s_1+s_2+s_3+s_4=9.6+2.8+2+1.7=16.1$.

Try $s_5=1.7$, $\therefore$ $l=16.1+0.8=16.9$, $\therefore$ $s=0.2\times8.4=1.7$, as assumed.

The total length now $=s_1+\ldots+s_5=17.8$, or 0.2 over the measured length to the crown. If the number of divisions is satisfactory, this small excess can be distributed amongst the several divisions in proportion to their length, with sufficient accuracy. Thus $0.20\div17.8=0.01$, whence we subtract from s_1, $9.6\times.01=0.10$; from s_2, $2.8\times.01=.03$; from s_3, s_4, and s_5, 0.02 each; giving the final values $s_1=9.50$, $s_2=2.77$, $s_3=1.98$, $s_4=1.68$, and $s_5=1.68$, whose sum is 17.61, which is within 0.01 of the measured length.

In case the difference is much greater than 0.2, the new lengths may be computed as above and used as trial lengths for a second "trial-and-error" computation, and the results again corrected as shown. A third trial is never needed.

Suppose, for example, that only four divisions of the semi-arch are desired; then using the first computed values, $s_1+s_2+s_3+s_4=16.1$ ft., which lacks

1.5 ft. of the length 17.6 ft. from crown to springing. The increase per foot is $1.5/16.1 = 0.093$. The increases of s_1, s_2, s_3, s_4 are thus: $9.6 \times .093 = 0.90$, $2.8 \times .093 = 0.26$, $2 \times .093 = 0.19$, and $1.7 \times .093 = 0.16$, respectively, giving for the new trial values of

$$\begin{array}{cccc} s_1, & s_2, & s_3, & s_4, \\ 10.49, & 3.06, & 2.19, & 1.86, \end{array}$$

respectively. At $\frac{1}{2}s_1 = 5.2$, $d^3 = 44$; hence this trial, $\frac{s_1}{d_1{}^3} = \frac{10.49}{44} = 0.24 =$ "constant."

$$\therefore \; s = 0.24d^3.$$

Proceeding exactly as above, by the "trial-and-error" method, we very quickly find

$$s_1 = 10.49, \quad s_2 = 3.00, \quad s_3 = 2.16, \quad s_4 = 1.97.$$

The sum is 17.62, practically exact, so that no further adjustment is required. It will be observed that these values are nearly those found above, so that the proportional method seems to give nearly accurate results, even in the case of large differences.

For the accurate computation of the stresses in an arch, the number of divisions of the semi-arch should be at least ten, and for very large arches, fourteen

or more. As the object, in the investigation of the arch shown in Fig. 11, is to illustrate methods, only four divisions (as just found) were used in order to make a clear figure and shorten the discussion.

In connection with reinforced arches, it may be observed that when a diagram has been prepared similar to Fig. 9, giving for any l the corresponding moment of inertia, the subsequent work is no longer than for plain concrete arches, as illustrated above. Also, the rough rule to ensure (about) a certain number of divisions is of utility.

Thus for the 50-foot span treated in the writer's "Theory of Steel-concrete Arches," at $\frac{l_c}{2}=\frac{29}{2}=14.5$, $d=1.35$ and $I=I_1+20I_2=0.256$. Hence if ten divisions of the semi-arch are desired, $l_c/10=29/10=2.9$.

$$\therefore \frac{l_c/n}{I}=\frac{2.9}{0.256}=11.3=\text{"constant."}$$

The actual final constant found was 11.5, which shows that the rule is a working one. It actually gives ten divisions in this instance.

Example.—Divide the semi-arch ring above, to satisfy the condition $s \div I=$constant, into 6 parts.

Although a diagram, Fig. 9, was used above for clearness, it is generally not necessary except perhaps for a reinforced arch, since the values of d^3 can be taken at once out of the tables, after the value of d has been scaled at the middle of the assumed division on the arch. It defaces

the drawing much less, however, to use a diagram, and it is advisable not to have too many holes pricked in the paper, particularly in the vicinity of points a.

It has been doubtless remarked that the values of s_1 are much larger than the value of s next the crown. When it is observed, however, that the moment of inertia at the spring is nearly 16 times that at the crown, it is evident that the resistance to bending near the abutment is so much greater there than near the crown, that we should expect the elastic curve and pressure line to be determined mainly by the upper part of the arch.

It may be objected that in the applications of the elastic theory, the abutments are taken as inelastic, as well as the foundations, whereas both are elastic. True, but suppose an ordinary sized abutment is included as part of the arch ring (after rounding off corners, too); its moment of inertia is usually so much greater than that of the arch ring that its effect on distortion, and therefore on the pressure curve, is absolutely negligible. For a still stronger reason, the elasticity of a good foundation—especially one of rock—can be ignored. If the foundation is doubtful, perhaps a three-hinged arch should be built. If the abutment is very small—scarcely more than a continuation

of the arch—it should undoubtedly be treated as a part of the elastic arch.

If we suppose the radial depths of arch ring to approach equality everywhere, the lengths s_1, s_2, . . . approach equality. For a constant depth they are exactly equal.

COMPUTATION OF LOADS.

21. The neutral line of the symmetrical arch shown in Fig. 11 is assumed to be a circular arc with span 30 ft. and rise 8 ft. The radial depth of the arch ring at the spring is taken at 5 ft., at the crown 2 ft., and the earth filling is assumed to extend 3 ft. above the crown. Circular curves are drawn for the extrados and intrados equidistant from the neutral line.

The plain concrete arch is supposed to weigh 150 lbs. per cubic foot, the earth filling $\frac{2}{3}$ this amount, or 100 lbs. per cubic foot. The ordinates from the extrados to the upper limit of the backing are now reduced to $\frac{2}{3}$ the original length, so that the new upper limit corresponds to a filling of the same density as that of the arch ring.

Having laid off, along the neutral line, from the center of each springing the successive lengths $s_1=10.49$, $s_2=3$, $s_3=2.16$, $s_4=1.97$, found in the preceding article, up to the crown, the *mid-points* of

each division are found by dividers and marked in order from the right springing to the left, $a_1, a_2, \ldots, a_8$, as shown on the figure.

Vertical lines are now drawn through $a_1, \ldots, a_8$ and other points shown, limited by the reduced contour of the earth filling and the intrados, and the areas in square feet included between any two consecutive verticals are computed. Since the slice of the arch considered is one foot thick, these areas are also the volumes of the corresponding parts, and it is only necessary to multiply them by 150 to get the weights in pounds of the parts considered.

Where the divisions are narrow, the corresponding forces P are assumed to act along the dotted verticals, each midway between the consecutive full lines, limiting the divisions.

In any case the following well-known graphical construction will determine the center of gravity of the trapezoid (see Fig. 10).

Bisect the parallel sides AB and CD at E and F. Draw EF.

Produce BA a distance AI = CD = length of side parallel to BA. Produce DC a distance CH = AB.

Draw IH. Where it intersects EF is G, the center of gravity of the trapezoid ABCD.

All dimensions throughout should be read to hundredths of a foot. As the

space from a_1 to a_2 is rather large, a full vertical midway between a_1 and a_2 was drawn and the forces P_1, P_1' computed; also P_0, the weight from the vertical at a_1 to the vertical through the inner edge of the springing joint, was ascertained. Lastly, the weight vertically over the

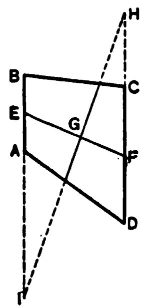

Fig. 10.

springing joint was found to be 5670 lbs.

The other weights in pounds are:

$$P_0 = 2176, \qquad P_2 = 1695,$$
$$P_1 = 3254, \qquad P_3 = 1276,$$
$$P_1' = 2434, \qquad P_4 = 612.$$

These are laid off from O vertically upwards, to scale of loads, in the order P_4, P_3, . . . , O′, the subscripts only being retained on "the load line."

For the left half a live load of 800 lbs. per square foot is assumed from the left abutment to the vertical through a_6. This corresponds to a locomotive load of 10,000 lbs. per linear foot, distributed through cross-ties and earth over 12½ ft.

The load P_5 from the crown to a_5 = load P_4 from crown to a_4. Similarly $P_6 = P_3$. The other dead loads must now be augmented by the live loads resting on the division, giving, in pounds,

$$P_7 = P_2 + 2.50 \times 800 = 3695;$$
$$P_8' = P_1' + 2.95 \times 800 = 4794;$$
$$P_8 = P_1 + 2.94 \times 800 = 5606;$$
$$P_9 = P_0 + 1.48 \times 800 = 3360.$$

Over spring $O = 5670 + 4.14 \times 800 = 8982.$

These loads are laid off, in the order $P_5, P_6, \ldots, O$, from the center of the left spring vertically downwards, thus completing the load line.

To lay off the loads accurately, their sum from the top downward should be computed and laid off to avoid errors being carried on.

The computer should not begin his work until he is provided with Barlow's "Table of Squares, Cubes, etc.," and Crelle's "Rechentafeln," the last-named work giving the product of three-figure numbers by similar numbers. Multiplications of any two numbers are made with ease

and accuracy. Certain divisions, too, are at once taken out; but the method is not so well adapted to division of large numbers. Of course a small slide rule is always serviceable, and Thacher's large one can take the place of the Rechentafeln. Logarithms are but rarely used in the computations.

CONSTRUCTION OF THE TRIAL EQUILIBRIUM POLYGON.

22. Let us assume a horizontal thrust at the crown, and draw from O, the division point between the segments 4 and 5 of the load line, a horizontal pole distance $= H' = 10,000$ lbs., to scale of loads, to the trial pole P′. For larger arches this pole distance can be made some multiple of 10,000, as 20,000, 30,000, etc. Having drawn the "rays" from P′ to the division points of the load line, draw from some point A, vertically beneath the crown, a horizontal to intersection with load P_4; then a line ∥ ray 34 to intersection with load P_3; then from this intersection with P_3, a line ∥ ray 23 to load P_2, and so on. To avoid mistake notice the relation that the line between P_3 and P_2 is ∥ ray 32, the subscript numbers being the same as the numbers on either side of the ray. To the left of A the same rule is observed. Thus from where the horizontal through A cuts P_5, draw line

to P_6 ‖ ray 56; from this point on P_6 to P_7, draw line ‖ ray 67, and so on.

As errors are carried on by this process, it is advisable to compute the ordinates from the horizontal h_8Ah_1 to the equilibrium polygon by the use of formula (1) of Art. 4:

$$m_s = m_r + R_r a.$$

The numerical application of this formula to the right half of the arch is as follows:

$$\begin{aligned}
m_3 &= P_4 \times 1.52 = 930;\\
m_2 &= m_3 + (P_4 + P_3)2.3 = 5272;\\
m_1' &= m_2 + (P_4 + P_3 + P_2)2.74 = 15{,}089;\\
m_1 &= m_1' + (P_4 + \ldots + P_1')2.95 = 32{,}839;\\
m_0 &= m_1 + (P_4 + \ldots + P_1)2.2 = 53{,}235.
\end{aligned}$$

For the left half we have

$$\begin{aligned}
m_6 &= P_5 \times 1.52 = 930;\\
m_7 &= m_6 + (P_5 + P_6)2.3 = 5272;\\
m_8' &= m_7 + (P_5 + P_6 + P_7)2.74 = 20{,}569;\\
m_8 &= m_8' + (P_5 + \ldots + P_8')2.95 = 51{,}181;\\
m_9 &= m_8 + (P_5 + \ldots + P_8)2.2 = 86{,}344.
\end{aligned}$$

The work can be checked by computing m_0 and m_9 independently, by taking moments of all the loads from the crown to P_0 for the right half, and similarly as to P_9 for the left half.

We now divide the moments above by $H' = 10{,}000$ to get the ordinates from the horizontal through A to the equilibrium polygon in feet. These ordinates

are now laid off to the scale of distance along the verticals P_4, P_3, etc. Lines through consecutive points thus found constitute the equilibrium polygon. It cuts the verticals through $a_1, a_2, \ldots$ at the points $b_1, b_2, \ldots$, and will be styled polygon b.* The object of the following procedures is to ascertain the amount of shifting and the true pole distance for such a trial polygon b, so that the changed polygon can be put in its true position on the arch as polygon $c_1, c_2, \ldots, c_8$, the true pressure line for the arch.

ESTABLISHING THE CLOSING LINE mm_1 OF POLYGON b.

23. The origin of co-ordinates is taken at O, x horizontal, y vertical. Thus the co-ordinates of a_1 are (x_1, y_1); of a_2 (x_2, y_2), etc.

The reason for the following steps will be given later (Art. 27). The "closing line" mm_1 is to be drawn so as to satisfy the conditions

$$\Sigma(mb) = 0, \quad \Sigma(mb \,.\, x) = 0,$$

where mb is an ordinate from mm_1 to points such as $b_1, b_2, \ldots, b_8$. mb is posi-

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* Any bh can also be accurately *computed* from the ordinates along the P's on either side. See Art. 39.

tive when b is above mm_1, negative when b is below mm_1.

We have, satisfying this convention as to signs, $mb = mh - bh$; therefore the preceding equations can be written

$$\Sigma(mh) = \Sigma(bh), \quad \Sigma(mh \cdot x) = \Sigma(bh \cdot x).$$

The sums in all these equations are supposed to include all the points b, as b_1, b_2, ..., b_8, for the entire arch.

Let the lines mh and bh be treated temporarily as forces acting vertically downwards. Their resultants are $\Sigma(mh)$ and $\Sigma(bh)$ respectively. If the first acts at a distance x_0 from O, the second x_0' from O, then it follows, from the last equation above, that

$$x_0 \Sigma(mh) = x_0' \Sigma(bh),$$

since the moment of the resultant is equal to the sum of the moments of the components.

In view of the preceding equation, we have $x_0 = x_0'$, or in words, *mm_1 must be so drawn that the resultant of the mh's, treated as forces, must equal and coincide with the resultant of the bh's.*

Let N = number of ordinates y, = number of a's, = number of b's, = 8 in the figure. Lay off $\mathrm{AD} = \dfrac{\Sigma(bh)}{\mathrm{N}}$ and draw any

trial closing line nn_1 through B, mm_1 being the true one. Then

$$\Sigma(nh)=\Sigma(mh)=\mathrm{N}\,.\,\mathrm{AD}=\Sigma(bh),$$

since N . AD = sum of ordinates v_1h_1, v_2h_2, . . . in rectangle $vv_1h_1h_8$, and since the segments vn added in the triangle $v\mathrm{D}n$ equal those subtracted in the triangle $v_1\mathrm{D}n_1$.

The first part of the condition is then fulfilled by either the line nn_1 or the true closing line mm_1, the resultants $\Sigma(nh)$ or $\Sigma(mh)$ and $\Sigma(bh)$ being all equal. In order that they may coincide, their moments (treating the bh's and mh's as forces) about AD must be equal, or

$$\Sigma(bh\,.\,z)=\Sigma(mh\,.\,z),$$

where z_1, z_2, z_3, . . . are the horizontal *distances* (always positive) from the vertical through A to b_1, b_2, b_3, . . . , respectively. Left-handed moments will be treated as positive. We can quickly compute

$$\Sigma(bh\,.z)=(b_8h_8-b_1h_1)z_1+(b_7h_7-b_2h_2)z_2+\ldots$$

by measuring to scale of distance b_8h_8, etc., tabulating the differences and effecting the products as illustrated in a following table.

Regarding now the line nn_1, we have

$$\Sigma(nh.z)=(nh_8-n_1h_1)z_1+(n_7h_7-n_2h_2)z_2+\ldots$$
$$=2(v_1n_1.z_1+v_2n_2.z_2+v_3n_3.z_3+v_4n_4.z_4).$$

Designate this last sum by $\Sigma(vn.z)$. If we lay off

$$v_1m_1=\frac{\Sigma(bh\,.\,z)}{\Sigma(vn\,.\,z)}\cdot v_1n_1, \quad . \quad . \quad (i)$$

then each vm is equal to the corresponding vn multiplied by $\Sigma(bh\,.\,z)\div\Sigma(bn\,.\,z)$; hence

$$\Sigma(vm\,.\,z)=\frac{\Sigma(bh\,.\,z)}{\Sigma(vn\,.\,z)}\Sigma(vn\,.\,z)=\Sigma(bh\,.\,z).$$

But $\Sigma(vm\,.\,z)$, by the notation above, $=\Sigma(mh\,.\,z)$.

$$\therefore\ \Sigma(mh\,.\,z)=\Sigma(bh\,.\,z).$$

The closing line mm_1 now satisfies the two conditions, $\Sigma(mh)=\Sigma(bh)$ and $\Sigma(mh\,.\,z)=\Sigma(bh\,.\,z)$, required, and is correctly established.

However, the work can be further simplified as follows:

From similar triangles,

$$\frac{z_1}{v_1n_1}=\frac{z_2}{v_2n_2}=\frac{z_3}{v_3n_3}=\ldots;$$

$$\therefore\ \Sigma(vn.z)=2\frac{z_1}{v_1n_1}(v_1n_1.z_1+v_2n_2.z_2+\ldots)\frac{v_1n_1}{z_1}$$
$$=2(z_1^2+z_2^2+z_3^2+z_4^2)\frac{v_1n_1}{z_1}.$$

This substituted in (i) above gives

$$v_1 m_1 = \frac{\Sigma(bh.z)}{2(z_1{}^2 + z_2{}^2 + \ldots)} . z_1 = \frac{\Sigma(bh.z)}{\Sigma(z^2)} . z_1. \quad (ii)$$

The denominator $\Sigma(z^2) = 2(z_1{}^2 + z_2{}^2 + \ldots)$ designates, for any arch, twice the sum of the squares of the z's to one side of the crown, or the sum for all the points b for the entire arch.

24. *Closing Line mm_1 for Symmetrical Loading.*—If the symmetrical arch is loaded with its own weight only, or, in addition, with a live load symmetrically distributed with respect to the crown, the equilibrium polygon b is symmetrical with respect to AD; hence $\Sigma(bh.z) = 0$, and by (ii) $v_1 m_1 = 0$.

Therefore we lay off, as before,

$$\text{AD} = \frac{\Sigma(bh)}{\text{N}},$$

and through D draw vv_1 parallel to $h_8 h_1$, to represent the true closing line. By summing up the values of bh to one side of the crown and dividing by $\frac{1}{2}$N, we reach more quickly the value of AD.

25. *Location of Line kk_1 on Arch.*—The line kk_1 is to be located in a similar manner with respect to the points $a_1, a_2, \ldots$ on the neutral line; but since the lengths

of the ordinates y are known, it is more convenient to proceed as follows:

Measure the ordinates $y_1, y_2, \ldots$ from OO′ to $a_1, a_2, \ldots, a_8$, and divide their sum by their number, to find the distance e from OO′ to kk_1, or

$$e = \frac{\Sigma(y)}{\mathrm{N}}.$$

Call $\Sigma(e) = \mathrm{N}e = \Sigma(y)$;

$$\therefore\ \Sigma(y-e)=0, \quad \text{or} \quad \Sigma(ka)=0.$$

Also from symmetry, $\Sigma(ka\,.\,x)=0$.

Hence the two conditions, $\Sigma(ka)=0$, $\Sigma(ka\,.\,x)=0$, for fixing the line kk_1 are fulfilled, ordinates ka referring to vertical ordinates from kk_1 to $a_1, a_2, \ldots$, ordinates above kk_1 being regarded as positive, those below as negative.

LOCATING THE TRUE EQUILIBRIUM POLYGON $c_1, c_2, \ldots$ ON THE ARCH.

26. It is now necessary to compute, for the whole arch,

$$\Sigma(ka\,.\,y) = \Sigma(y-e)y = \Sigma(y^2) - e\Sigma y;$$

$$\therefore\ \Sigma(ka\,.\,y) = \Sigma(y^2) - \frac{(\Sigma y)^2}{\mathrm{N}},$$

or the sums can be taken for the half-arch and doubled.

A similar sum, $\Sigma(mb \,.\, y)$, must also be made out for the entire arch, mb representing a vertical ordinate from mm_1 to points $b_1, b_2, \ldots$, regarded as positive when b is above mm_1, negative when b is below mm_1.

Since $mb = mh - bh$,

$$\therefore\ \Sigma(mb \,.\, y) = \Sigma(mh \,.\, y) - \Sigma(bh \,.\, y).$$

Now in Fig. 11

$$\Sigma(mh.y) = (m_1h_1 + mh_8)y_1 + (m_2h_2 + m_7h_7)y_2 + \ldots$$
$$= 2\mathrm{AD}(y_1 + y_2 + y_3 + y_4) = \mathrm{AD}\Sigma(y).$$

$$\therefore \Sigma(mb.y) = \frac{\Sigma(bh)}{\mathrm{N}}\Sigma(y) - \Sigma(bh.y). \quad . \quad . \quad . \quad (iii)$$

(All of these sums are entered in the table, Art. 28, p. 57.)

It is a principle of the equilibrium polygon, Art. 5, that if the ordinates mb are altered in a given ratio, the pole distance is altered in the inverse ratio. The ordinates mb are now altered in the ratio $\dfrac{\Sigma(ka \,.\, y)}{\Sigma(mb \,.\, y)}$, and the assumed pole distance in the inverse ratio.

The new ordinates mb are now laid off vertically above or below kk_1, according to sign, to locate all the points $c_1, c_2, \ldots$ in the same verticals with $a_1, a_2, \ldots$, re-

spectively. The old ordinates mb have thus been changed to kc. Similarly, AD can be altered in the given ratio and laid off above kk_1 to give the center of pressure at the crown. The proof for the step above and that which follows has been given in Art. 5.

To find the true pole P, draw from the trial pole P′ a parallel to mm_1 to intersection with the load line at I, then horizontally to the right a distance IP = assumed pole distance $\times \dfrac{\Sigma(mb \,.\, y)}{\Sigma(ka \,.\, y)}$ = true pole distance measured to the scale of loads. This locates P, the true pole, and the true pole distance, to scale of loads thus found is the horizontal thrust of the arch.

Beginning at the center of pressure at the crown, the equilibrium polygon c can be tested by the usual graphical construction, regarding P as the pole (Art. 2).

DEMONSTRATION.

27. Since the equilibrium polygon b, with ordinates all altered in the same ratio, is now in position on the arch as polygon c, the closing line mm_1 now coinciding with kk_1 and any ordinate mb having been changed to the corresponding kc, we have, from Art. 23,

$$\Sigma(kc) = 0. \qquad \Sigma(kc \,.\, x) = 0.$$

Also, it was shown in Art. 25 that the line kk_1 was located to satisfy the conditions

$$\Sigma(ka)=0, \qquad \Sigma(ka\,.\,x)=0,$$

the summation extending over the entire arch as before.

On subtracting the last equations from the preceding, we have

$$\Sigma(ac)=0, \qquad \Sigma(ac\,.\,x)=0.$$

Hence the construction ensures that the first two conditions, Art. 19, p. 33, for an arch without hinges, shall be fulfilled. The third condition is also fulfilled, for by Art. 26, p. 53, we laid off

$$kc=\frac{\Sigma(ka\,.\,y)}{\Sigma(mb\,.\,y)}mb.$$

We can write down N equations of this type on giving proper subscripts to kc and mb, $_{1,\,2,\,3,}$ .... Now multiply each equation with the corresponding y, and add

$$(kc_1\,.\,y_1+kc_2\,.\,y_2+\ldots)=\frac{\Sigma(ka\,.\,y)}{\Sigma(mb\,.\,y)}(mb_1\,.\,y_1+mb_2\,.\,y_2+\ldots).$$

Or, since $(kc_1\,.\,y_1+kc_2\,.\,y_2+\ldots)=\Sigma(kc\,.\,y)$ and

$$(mb_1\,.\,y_1+mb_2\,.\,y_2+\ldots)=\Sigma(mb\,.\,y),$$

this equation reduces to

$$\Sigma(kc\,.\,y) = \Sigma(ka\,.\,y).$$

$$\therefore\ \Sigma(kc - ka)y = 0. \qquad \therefore\ \Sigma(ac\,.\,y) = 0.$$

Therefore the third condition of Art. 19, p. 33, is also fulfilled by the equilibrium polygon *c*. It thus satisfies all the conditions for an arch fixed at the ends and without hinges, and is thus the true one.

RÉSUMÉ OF OPERATIONS.

28. Let us give now a résumé of operations applicable to any arch, though the numerical values given refer only to the arch of Fig. 11.

After the neutral line is divided so that s/I is constant (Art. 20), the points *a* marked at the middle of each division, the weights P found (Art. 21), and the equilibrium polygon *b* drawn (Art. 22), we proceed as follows:

(1) Make out a table of quantities, as given below, by measuring *y*, *z*, and *bh*, to the scale of distance, to hundredths of a foot.

(2) Lay off $\mathrm{AD} = \frac{\Sigma(bh)}{\mathrm{N}} = \frac{15.05}{8} = 1.88.$

N = number of *a*'s or number of *b*'s for the entire arch. $\Sigma(bh)$, as computed, may be checked by laying off along a

QUANTITIES FOR ARCH OF 30-FT. SPAN, 8-FT. RISE (FIG. 11).

1	2	3	4	5	6	7	8	9
Point.	y.	y^2.	z.	z^2.	bh_R.	bh_L.	(Col. 7 − col. 6)z.	(Col. 6 + col. 7)y.
a_1	$y_1 = 3.89$	15.13	$z_1 = 11.48$	131.8	$b_1h_1 = 4.67$	$b_8h_8 = 7.53$	$2.86 \times z_1 = 32.89$	$12.2 \times y_1 = 47.46$
a_2	$y_2 = 7.13$	50.84	$z_2 = 5.60$	31.4	$b_2h_2 = 0.98$	$b_7h_7 = 1.23$	$0.25 \times z_2 = 1.40$	$2.21 \times y_2 = 15.76$
a_3	$y_3 = 7.73$	59.75	$z_3 = 3.10$	9.6	$b_3h_3 = 0.30$	$b_6h_6 = 0.30$	$0 \times z_3 = 0.00$	$0.60 \times y_3 = 4.64$
a_4	$y_4 = 7.98$	63.68	$z_4 = 1.02$	1.0	$b_4h_4 = 0.02$	$b_5h_5 = 0.02$	$0 \times z_4 = 0.00$	$0.05 \times y_4 = 0.40$
	26.73 2	189.40 2		173.8 2	5.97	9.08 5.97	$\Sigma(bh \cdot z) = 34.29$ $= \Sigma(bh_L - bh_R)z$	$\Sigma(bh \cdot y) = 68.26$ $= \Sigma(bh_R + bh_L)y$
	$\Sigma(y) = 53.46$	378.80 $= \Sigma(y^2)$		347.6 $= \Sigma(z^2)$		$\Sigma(bh) = 15.05$		

The symbol Σ refers throughout to sums taken for the *entire* arch, whether with reference to points a or points b. All lengths, as y, z, or bh, must be carefully measured or computed to the hundredth of a foot.

Note that $\Sigma(bh \cdot z)$ involves the idea of taking moments of the bh's about AD. Its value is $\Sigma(bh_L - bh_R)z$.

line, with dividers, b_1h_1, b_2h_2, . . . , and measuring to scale of distance. Designate ordinates bh to left and right of the crown bh_L and bh_R respectively.

(3) Lay off $vm = v_1m_1 = \frac{\Sigma(bh_L - bh_R)z}{\Sigma(z^2)} z_1$

$$= \frac{34.29 \times 11.5}{347.6} = 1.13.$$

Draw mm_1.

(4) Compute $e = \frac{\Sigma(y)}{N} = \frac{53.46}{8} = 6.68.$

Draw kk_1 parallel to OO′ and a distance e above it.

(5) Compute $\Sigma(ka \,.\, y) = \Sigma(y^2) - \frac{(\Sigma y)^2}{N} = 378.8 - \frac{(53.46)^2}{8} = 21.5.$

(6) Compute $\Sigma(mb \,.\, y) = \frac{\Sigma(bh)}{N} \Sigma(y) - \Sigma(bh \,.\, y) = 1.883 \times 53.46 - 68.26 = 32.40.$ (See (2) above and column 9 of table.)

(7) Lay off above or below kk_1, according as mb is positive or negative,

$$kc = \frac{\Sigma(ka \,.\, y)}{\Sigma(mb \,.\, y)} \,.\, mb = \frac{21.5}{32.4} mb = 0.664mb,$$

and thus find kc_1, kc_2, . . . , and establish the points c_1, c_2, . . . , and the center of pressure at the crown.

(8) H′ being the assumed horizontal thrust for polygon b (10,000 lbs. in the example), then the true horizontal thrust for polygon c or for the arch is

$$H = \frac{\Sigma(mb\ .\ y)}{\Sigma(ka\ .\ y)} H' = \frac{32.4}{21.5} 10{,}000 = 15{,}100 \text{ lbs.}$$

(9) Draw P′I parallel to mr_1 from the trial pole P′ to intersection I with the load line; then draw IP = H (= 15,100) to scale of loads, horizontally to the right, to locate P, the true pole. With P as the pole, the upper ray of the force diagram is the reaction at the right spring, the lower ray the reaction at the left spring. The point I divides the load line into the two vertical components of the reactions. As a check, having the center of pressure at the crown and true pole P, the equilibrium polygon c can be drawn. It should coincide with the polygon c_1, c_2, . . ., already established.

29. To find the centers of pressure on the springing sections, we simply prolong the polygon c to those sections.

Thus drawing the rays 01, 0O′, O′C′ to the true pole P, a line from c_1 parallel to ray 01 is drawn to intersection with P_0; thence a line ∥ ray 0O′ to intersection with vertical through center of gravity of dead load vertically over right springing section; thence a line ∥ ray O′C′

to intersection with springing section—the center of pressure on that section.

Similarly at left draw from c_8 a line $\parallel$ ray 89 to vertical P_9; thence a line $\parallel$ ray 9O to vertical through center of gravity of dead and live load over left springing section; thence a line $\parallel$ ray OC to intersection with springing section, which is thus the center of pressure on that section.

PROPER POSITION OF LOADS P RELATIVE TO POINTS *a*.

30. By Arts. 13 and 17 it was premised that $M = Ht$ should be accurately found at the middle of $s_1, s_2, s_3, \ldots$, or at $a_1, a_2, a_3, \ldots$, which is neatly effected by the construction of polygon c, the loads P having the position shown. Thus at a_3, $M = H \,.\, a_3c_3$ exactly.

If the true equilibrium *curve*, for infinitesimal divisions, be supposed drawn, it will fall below the equilibrium polygon c except at $a_1, a_2, \ldots$ (which are points on it), and the greatest departure is at a load P. Some authors use such a division of the arch that t has to be measured at a load P, which is a most unfortunate selection and leads to values of t as far removed from the true as possible.

For the upper part of the arch, where

the forces P are close together, this objection loses weight; but the position of the loads given on the figure is urged because it leads to theoretically exact results throughout.

Where the loads are transmitted down vertical columns resting on the arch ring, the equilibrium polygon has necessarily to be drawn for the forces as they are distributed. After drawing polygon *b*, however, if the angular points occur at any of the *b*'s, they should be slightly rounded off, for none such occur in reality for either polygons *b* or *c*. The equilibrium *polygon*, as we draw it, is not the true pressure line. The latter is always a *curve*, corresponding to infinitesimal divisions and loads, for either arch ring with earth backing or with columns.

31. *Case where* $(\mathrm{I}_1+n\mathrm{I}_2)$ *Varies as the Secant of the Inclination* i *of the Arc to the Horizontal.*—If the horizontal projection of each division s of the neutral line is constant and equal to $2h$, then $s \div (2h) = \sec i$ at the corresponding a. Then the condition of Art. 19, that $s \div (\mathrm{I}_1+n\mathrm{I}_2) = \text{constant}$, can be expressed on dividing by $2h$. $\sec i \div (\mathrm{I}_1+n\mathrm{I}_2) = \text{constant}$, or $(\mathrm{I}_1+n\mathrm{I}_2)$ must vary as $\sec i$.

As this condition is so rarely fulfilled in modern designs, I shall simply give a consequence without the demonstration. In Fig. 11, if $n =$ number of a's or b's to the right of the crown, and $z_4 = h$, then $z_3 = 3h$, $z_2 = 5h$, $z_1 = (2n-1)h$. Also, let $\Delta_1 = b_5h_5 - b_4h_4$, $\Delta_2 = b_6h_6 - b_3h_3$, $\Delta_3 = \text{etc.}$,

the ordinates subtracted being equally distant from the crown, Δ_1 corresponding to those nearest the crown; then it can be shown that

$$vm=\frac{3}{2}\cdot\frac{\Delta_1+3\Delta_2+5\Delta_3+7\Delta_4+\ldots}{n(2n+1)}.$$

In this case the *span* OO′ is divided into $2n$ equal divisions, and perpendiculars erected at the middle of each, whose intersections with the neutral line give the points *a*.

ALGEBRAIC SOLUTION.

32. The vertical component V of the thrust at the crown is given by IO to the scale of loads. From similarity of the triangles IOP′ and mvD, we have

$$\mathrm{V}=\frac{vm}{z_1}\mathrm{H}'=\frac{\Sigma\{(bh_L-bh_R)z\}}{\Sigma(z^2)}\mathrm{H}', \quad (iv)$$

on using the value of vm given by (3), Art. 28.

Also, by use of the values of $\Sigma(ka\,.\,y)$ and $\Sigma(mb\,.\,y)$, given in Art. 28, (5) and (6), we find, since

$$\mathrm{H}=\frac{\Sigma(mb\,.\,y)}{\Sigma(ka\,.\,y)}\,.\,\mathrm{H}',\quad kc=\frac{\Sigma(ka\,.\,y)}{\Sigma(mb\,.\,y)}\,.\,mb,$$

$$\mathrm{H}=\frac{\Sigma bh\Sigma y-\mathrm{N}\Sigma(bh\,.\,y)}{\mathrm{N}\Sigma(y^2)-(\Sigma y)^2}\mathrm{H}', \quad . \quad (v)$$

$$kc=\frac{\mathrm{N}\Sigma(y^2)-(\Sigma y)^2}{\Sigma bh\Sigma y-\mathrm{N}\Sigma(bh\,.\,y)}mb. \quad . \quad (vi)$$

The latter formula also applies to finding kc for the crown joint, when for mb we put $AD = \Sigma(bh) \div N$. Thus the formulas (*iv*), (*v*), (*vi*) determine completely the thrust and center of pressure at the crown, and therefore polygon *c*. Also calling $H' . bh = m$, (*iv*) and (*v*) can be reduced to forms similar to those given for V and H_0 by Turneaure and Maurer in "Reinforced Concrete Construction," p. 268, having regard for the difference in notation. The ordinate y being counted from the spring in the one case and from the crown in the other, the formulas for H are not identical.

Where great accuracy is required, as in a steel arch, then the values of z can be measured and y computed to thousandths of a foot, the ordinates *bh computed* to any desired accuracy, and polygon *b* drawn only as a rough check on the numerical work. Also, the value of $e = (\Sigma y) \div N$ is computed, and by (*iv*), (*v*), (*vi*) we find V, H, and kc for the crown, giving the center of pressure there. The bending moments at any point of the arch can now be computed. As a check on this purely algebraic computation, the lines mm_1, kk_1 may be drawn, and by (*vi*) the values of kc_1, kc_2, ... computed and laid off. The bending moment at any point *c* is then $H . ac$.

The force diagram, in connection with polygon *c*, gives most easily the normal thrust and shear on any section.

33. To illustrate the working of the method, the previous example will be taken and the numerical values of the table and of Art. 28 used, the results being given to only three significant figures.

Thus by (*v*), $H=\frac{32.4}{21.5}10{,}000=15{,}100$ lbs.;

by (*iv*), $V=\frac{34.29}{347.6}10{,}000=987$ lbs.

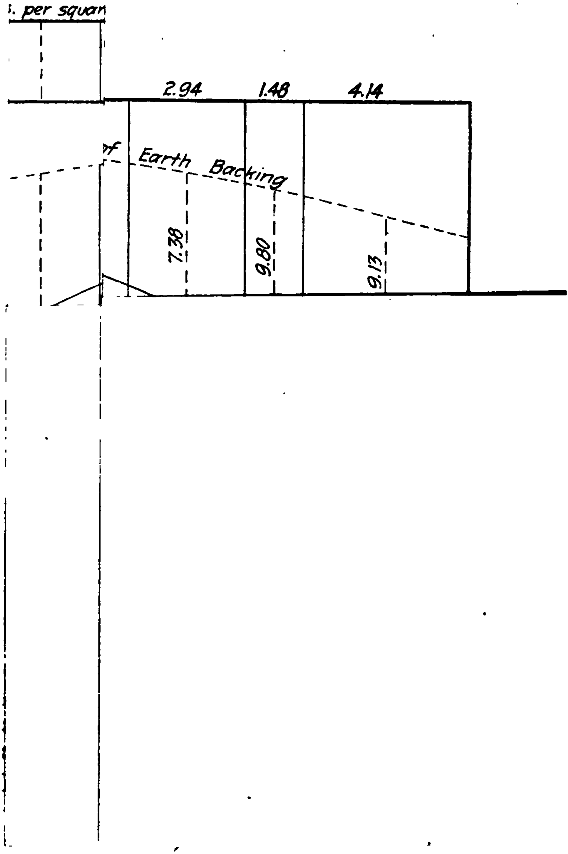
$$AD=\frac{\Sigma(bh)}{N}=\frac{15.05}{8}=1.88.$$

$\therefore$ by (*vi*), the value of kc for the crown is

$$kc=\frac{21.5}{32.4}\times 1.88=1.25.$$

Since $e=(\Sigma y)\div N=6.68$, the center of pressure at the crown is $6.68+1.25=7.93$ ft. above OO′, or $8.00-7.93=0.07$ ft. below the center of the joint, since the rise of the neutral line is 8 feet.

34. The bending moments at points *a* can be found in two different ways, the first involving the computation of a_1c_1, a_2c_2, . . . as follows:



. per squar
2.94
1.48
4.14
of Earth Backing
7.38
9.80
9.13

s *a*
the
c,

By Art. 28, (3),

$$\frac{v_1 m_1}{z_1} = \frac{34.29}{347.6} = 0.09865 = \frac{v_2 m_2}{z_2} = \text{etc.}$$

$v_1 m_1 = .09865\ z_1$; $v_2 m_2 = 0.09865\ z_2$; $v_3 m_3 =$ etc.; thus any vm can be computed.

Also, $$mh = \text{AD} \pm vm.$$

Since bh is supposed to be known by computation, and $mb = mh - bh$, any mb can be *computed.*

By (*vi*), $kc = 0.664mb$, from which any kc can be found.

Lastly, $$ac = e + kc - y.$$

The values of ac can thus be computed to any desired accuracy. The bending moment M at any point a of the arch is $\text{M} = \text{H} \cdot ac$.

Since from the preceding equation ac is positive when c is above a, negative when c is below a, the same is true for M. Hence with this convention it is seen that the bending moment causes the greatest compression at the extrados when M is positive, but the greatest compression at the intrados when M is negative. Also see Art. 16, note. It is an easy rule to remember that when the side of polygon c, pertaining to a section, passes above its center ($t = ac$ positive),

the bending moment $M=Ht$ causes compression at the extrados; if below, compression at the intrados, the greatest compression being on the edge of the section nearest polygon c, since the resultant on the section acts along the side pertaining to polygon c.

The second way of accurately computing bending moments is by considering the two halves of the loaded arch separately, each subjected to the loads and reactions and the forces H and V at the crown section. If V is positive it acts up for the left half, down for the right half. Of course H acts to the left in Fig. 12 (c), to the right in Fig. 12 (b); above the center of the crown section if $t_c=ac$ there is positive; below, when t_c is negative.

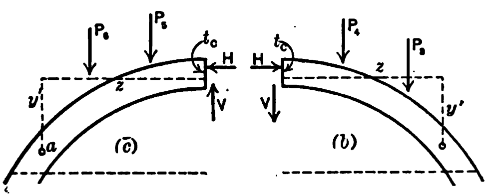


FIG. 12.

Let us call the horizontal and vertical distances from the center of the crown joint to a point a, z and y' respectively,

and m the moment of the loads from the crown to a about a.

Then for the left half we have

$$M = H(y' + t_c) - m + Vz,$$

taking left-handed moments as positive. At the crown $M_c = Ht_c$; left handed about the center there if t_c is positive, as drawn.

For the right half, right-handed moments will be taken as positive.

$$M = H(y' + t_c) - m - Vz.$$

For either half, according as M is positive or negative, the greatest compression is at the extrados or intrados respectively; otherwise polygon c passes above or below a according as M is positive or negative. After $M = Ht$ has been computed, on dividing by H, $t = ac$ for the section can be found and laid off, thus establishing the points c.

ARCH LOADED ONLY WITH ITS OWN WEIGHT.

35. The same 30-ft. span, Fig. 11, will now be treated for dead load only. The part of the equilibrium polygon required, $b_1, \ldots, b_4$, has already been drawn.

Then, as shown in Art. 24, we lay off $AB=\frac{5.97}{4}=1.49$ and draw mm_1 through B parallel to Ah_1. We derive $5.97=\Sigma(bh_R)$ from column 6 of the table of Art. 28. We have

$$\Sigma(mb\,.\,y)=\frac{\Sigma bh}{N}\,.\,\Sigma y-\Sigma(bh\,.\,y),$$

$$=1.49\times 53.46-2\left\{\begin{array}{l}4.67\times 3.89\\+.98\times 7.13\\+.30\times 7.73\\+.02\times 7.98\end{array}\right\}=24.38.$$

Hence, since the closing line kk_1 is unchanged, as before $\Sigma(ka\,.\,y)=21.5$;

$$\therefore\ \Sigma(ka\,.\,y)\div\Sigma(mb\,.\,y)=21.5\div 24.38$$
$$=0.882.$$

$\therefore\ kc=0.882mb$, from which the points $c_1,\ldots,c_4$ are located. They are not distant from the corresponding a's more than 0.03 ft. The horizontal thrust is now

$$\frac{24.38}{21.5}10{,}000=11{,}300 \text{ lbs.}$$

This is laid off, to scale of loads, from O along horizontal OP′ to true pole.

In this arch, where the surcharge has been reduced considerably in height, the

pressure line from the crown to point a_1 coincides nearly with the neutral line. Below a_1 it rises above it, receding from it as we go downwards.

36. A word may be said here relative to accuracy where graphical constructions are resorted to. Straight edges on rulers and triangles are absolutely imperative. A steel ruler and an aluminum triangle are best for accuracy, though they soil the paper. With the usual scales of 2 ft. or 4 ft. to the inch, errors of at least 0.01 ft. may be looked for in locating points. Errors of 0.01 or 0.02 ft. may be made in drawing lines through the (needle hole) points. Thus without great care ordinates measured from such lines can easily be 0.02 ft. wrong.

Every new line that has to be drawn involves some error; hence it is advisable to introduce as few lines as possible. It will have been observed that in the table of Art. 28, the only quantities introduced are y, z, and bh, which should reduce error to a minimum.

In "Theory of Voussoir Arches," by the writer, pp. 173–5, it was found for the examples given, introducing other quantities than bh, y, and z in the computation, that nevertheless points c could possibly be counted on as within 0.05 ft. of their correct position. Certainly

we ought to attain greater accuracy by the new construction. In fact, the main error to be feared in the latter case is in drawing the polygon *b*. If the ordinates *bh* are all computed, the error due to inaccurate construction of polygon *b* is entirely eliminated.

Where the forces P are supposed to act along the medians of the corresponding trapezoids, slight errors are introduced; but they can be reduced either by increasing the number of divisions of the arch or by finding the center of gravity of each trapezoid.

The largest error is unavoidable and results from our not knowing beforehand the specific gravity of either the concrete ring or that of the earth filling.* The weight of concrete of the usual proportions varies from 140 to 150 lbs. per cubic foot. The average may be taken at 145, or if reinforced, say, 150 lbs. per cubic foot; but a well-compacted concrete will weigh more when originally placed as a dry mixture than where much water is used.

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* As to the weight of earth per cubic foot, Trautwine says that common loam, dry and loose, will weigh 72 to 80 lbs., but if moderately rammed or packed, 90 to 100 lbs. Quartz sand, dry and loose, may be put at 90 to 106 lbs., or even 117 lbs. if the sand is of pure quartz with very large and very small grains. The first figures are increased 12% by ramming. Water necessarily adds to the weight of compacted earth.

With regard to the live load transmitted to the arch through cross-ties and earth, it is certain that the earth distributes this load over a greater area than the base of the cross-tie, but exactly over what area, it is difficult to say.

CHAPTER III.

THREE-CENTERED ARCH.

ALGEBRAIC SOLUTION IN FULL.

37. We have seen, for the arch previously examined, that for dead load only, the pressure line coincides very nearly with the neutral line from the crown to a_1 and then departs more and more from it as we go downwards, always lying above the neutral line. It follows for the upper part of an arch, particularly when the filling is light, that the circular arc is a very satisfactory curve. For the lower portion it would seem, at first sight, that the curve should become flatter; but exactly the contrary is found to be the case for the intrados at least, in order to give a large springing joint to ensure that for ordinary live and dead loads no tension shall occur there.

In Fig. 13 is shown the left half of a three-centered arch, the radius of the upper portion DH of the intrados being R; for the lower portion AD, the radius

$=r=\mathrm{AI}=\mathrm{DI}$. If a and h are the half-span and rise of the upper portion, we have, by geometry,

$$(2\mathrm{R}-h)h=a^2; \quad \therefore \ \mathrm{R}=\frac{1}{2}\left(\frac{a^2}{h}+h\right).$$

Drop the perpendicular DF upon the span line and draw AB perpendicular to DI.

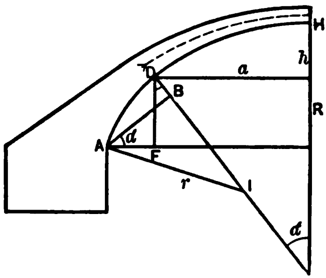

Fig. 13.

Then calling the half central angle of the upper portion α, we have

$$\mathrm{AD}^2=\mathrm{AF}^2+\mathrm{FD}^2,$$

$$\mathrm{BD}=\mathrm{FD}\cos\alpha-\mathrm{AF}\sin\alpha,$$

$$2r\,.\,\mathrm{BD}=\mathrm{AD}^2;$$

$$\therefore \ r=\frac{1}{2}\frac{\mathrm{AD}^2}{\mathrm{BD}}=\frac{1}{2}\frac{\mathrm{AF}^2+\mathrm{FD}^2}{\mathrm{FD}\cos\alpha-\mathrm{AF}\sin\alpha}.$$

When the span and rise are known and a and h assumed, AF and FD are known, so that R and r can be computed.

A number of arches have been built for highway traffic, assuming $a = \frac{1}{4}$ span and $h = \frac{1}{6}$ rise. Here the depth of key was made small, that at D about 25% greater, and at the springs the radial depth was assumed from two to three times that at the key for flat arches, to six times as much for a rise $\frac{1}{6}$ the span.

For arches of usual spans carrying a very heavy locomotive load, the proportionate depth at key must be greater—considerably greater—than for highway bridges. For such arches h has been increased even to $\frac{1}{2}$ the rise and a to $\frac{4}{5}$ the half-span. Only trial can determine the best proportions between the extreme limits given.

38. In the three-centered arch, Fig. 15, to be examined, the clear span was 90 ft., the rise 18 ft. Assuming $a = \frac{4}{5}\,\frac{90}{2} = 36$ and $h = 9.5$, the formulas give $R = 72.97$, $\alpha = 29°\ 33.7'$, $r = 26.88$.

The intrados can, of course, be drawn without this computation, as it is easy to find the radii R and r by drawing perpendiculars to chords DH and AB, Fig. 13, at their mid-points to intersections

with HG and DI respectively, to find the centers corresponding.

The depth of key here was assumed at 2.75 ft.; at O, 6 ft., and a circular curve of constant radius was drawn for the extrados. The neutral line is midway between the intrados and extrados and is practically given by two circular curves of different radii for the portions OD and DH.

The portion of the arch, Fig. 15, from O to O′ was assumed to rest on rigid abutments, since the portions below O and O′, although strictly elastic, can affect, on account of size, the position of the pressure line very slightly.

The neutral line from either O or O′ to the crown was divided into 14 divisions, so that s/I was constant for each division, after the method so fully given in Art. 20.

The length of neutral line l from O to crown was $l=46.33$ ft., and the radial depth at mid-length was 3.4 ft.; $\left(\frac{l}{n}\right)\div d^3$ $=\frac{3.31}{3.4^3}=0.084$. Hence the rough rule of Art. 20 would give $s=0.084d^3$. Actually, $s=0.09d^3$ was used and led to $n=14$ divisions of each half-arch.

The points a_1, a_2, . . . , a_{28} are at the middle of the corresponding divisions.

As usual, verticals are drawn through these points from the intrados to the reduced contour of the earth filling. The filling was assumed to weigh 100 lbs., the concrete arch ring 150 lbs. per cubic foot. The areas of the trapezoids between the verticals were computed and multiplied by 150 to give the dead loads P_1, P_2, etc. A live load of 750 lbs. per square foot was supposed to extend from the left abutment to a_{20}, or about three-eighths of the span OO′ of the center line. The weights corresponding were added to the dead loads to give the loads P_{21}, P_{22}, etc.

The vertical distances y above OO′ for each point a were measured; also the horizontal distances z, corresponding, from the crown, and the distances between consecutive P's. All of these quantities are entered in the tables following.

39. The moment m_s of all loads from the crown up to load P_s, about P_s, was computed from the formula

$$m_s = m_r + R_r a,$$

after the method fully explained in Arts. 4 and 22.

It is convenient to tabulate results as follows:

P (lbs.).	R = ΣP.	a feet.	aR.	m (foot-pounds).
$P_{15}=P_{14}=$ 724	724	1.39	1,006	$m_{13}=m_{16}=$ 1,006
$P_{16}=P_{13}=$ 1,405	2,129	1.86	3,960	$m_{12}=m_{17}=$ 4,966
$P_{17}=P_{12}=$ 1,482	3,611	1.89	6,824	$m_{11}=m_{18}=$ 11,790
$P_{18}=P_{11}=$ 1,550	5,161	1.98	10,219	$m_{10}=m_{19}=$ 22,009
$P_{19}=P_{10}=$ 1,621	6,782	2.00	13,564	$m_{9}=m_{20}=$ 35,573
$P_{20}=P_{9}=$ 1,722	8,504	2.07	17,603	$m_{8}=m_{21}=$ 53,176
$P_{8}=$ 1,803	10,307	2.15	22,160	$m_{7}=$ 75,336
$P_{7}=$ 2,060	12,367	2.43	30,052	$m_{6}=$ 105,388
$P_{6}=$ 2,440	14,807	2.62	38,794	$m_{5}=$ 144,182
$P_{5}=$ 2,940	17,747	3.05	54,128	$m_{4}=$ 198,310
$P_{4}=$ 3,652	21,399	3.60	77,036	$m_{3}=$ 275,346
$P_{3}=$ 4,887	26,286	4.30	113,030	$m_{2}=$ 388,376
$P_{2}=$ 6,984	33,270	6.00	199,620	$m_{1}=$ 587,996
$P_{1}=$ 12,924	46,194	6.36	293,794	$m_{0}=$ 881,790
$P_{0}=$ 13,680	59,874	1.38	82,626	At O′, $m=$ 964,416
59,874		43.08	964,416	

In this table the sum of the loads in the first column down to any line is entered in the second column on the same line. The distance a between any P and the next following is entered in the third column opposite the first-named P. The product of a and R is found in the column headed aR. In the last column the successive sums of the preceding column are recorded.* Thus, by the formula,

$$m_{11}=m_{12}+(P_{14}+P_{13}+P_{12})1.89=4966+6824=11,790, \text{ etc.}$$

As a part of this table applies to the left half until the live load is reached, the following table gives only the quantities for the part of the arch under the live load for the left half.

The values of m at O and O′ should be checked by taking moments of the P's from the crown about those points. This checks the whole work. Assuming for the trial polygon the thrust at the crown to be horizontal and equal to 100,000 lbs., we find the ordinates of the trial equilibrium polygon *b at the loads* by pointing off five decimal figures from the values of m given in the last columns.

* It will be observed that the sum of the numbers in the first column should equal the last number in the second column. Similarly for the last two columns. This checks the additions, but not the products aR.

Thus the ordinate at P_{24}, from $h_{28}Ah_1$ to the polygon b, is 1.69 ft., which is laid off to the scale of distance. The ordinates for O and O′, the centers of the springs, are likewise computed. They are 13.70 and 9.64 ft. respectively.

P (lbs.).	R = ΣP.	a ft.	aR.	m (foot-pounds).
P_{20} = ...	8,504	....	....	m_{21} = 53,176
P_{21} = 3,363	11,867	2.15	25,514	m_{22} = 78,690
P_{22} = 3,747	15,614	2.43	37,942	m_{23} = 116,632
P_{23} = 4,337	19,951	2.62	52,272	m_{24} = 168.904
P_{24} = 5,070	25,021	3.05	76,314	m_{25} = 245,218
P_{25} = 6,074	31,095	3.60	111,942	m_{26} = 357,160
P_{26} = 7,789	38,884	4.30	167,201	m_{27} = 524,361
P_{27} = 10,569	49,453	6.00	296,718	m_{28} = 821,079
P_{28} = 18,309	67,762	6.36	430,966	m_{29} = 1,252,045
P_{29} = 17,955	85,717	1.38	118,289	O, m = 1,370,334

77,213	1,317,158
8,504	53,176
85,717	1,370,334

To find the points $b_1, b_2, \ldots, b_{28}$, the polygon is drawn through the points just found, and the intersections with the verticals through $a_1, a_2, \ldots, a_{28}$ give the points b. As it is desirable to ascertain the values of bh within about 0.01 ft., it is necessary either to magnify the ordinates, say 10 times, or, better, to avoid too large a drawing, plot the polygon b on cross-section or profile paper to such a scale that bh can be measured accurately to within 0.01 ft.

It is, however, not necessary in this algebraic solution to lay off polygon b to scale on the drawing at all, except for purposes of theory. For subsequent positions of the live load it was not redrawn, as no measurements were taken from it. The following is suggested as the best practical solution.

To find any bh graphically, the ordinates at the P's must be plotted on cross-section or profile paper to such a scale that the ordinates, as well as $bh=b$, can be easily read to 0.01 ft. Thus for b_1, since P_1 and P_0 are 6.36 ft. apart and b_1 is distant from P_1 3.53 ft., lay off at P_1 the ordinate 5.88 and at P_0 (6.36 ft. from P_1) the ordinate 8.82, as taken from the table. Then connect the extremities with a straight line and read the ordinate b_1 (3.53 ft. from P_1) to scale. It is found to be 7.51 ft.

The same portion of the cross-section paper can thus be used over and over again, starting always at a full line.

The ordinates $b_1, b_2, \ldots, b_{28}$ are thus quickly found, say to within 0.01 ft., to scale. Practically for about 6 ordinates either side of the crown, any bh is the mean of the adjacent ordinates along the P's.

If bh is determined only to the nearest hundredth, we cannot expect a greater

accuracy for ac. Consequently if it should be considered desirable, as in a steel arch, say, to find ac to 0.001 ft., then bh should be *computed* to at least 0.001 ft. This may be done either by proportional triangles or from the formula for m above, on replacing a by d, the distance from the center of moments to the nearest P on

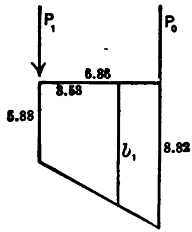

FIG. 14.

the crown side. Thus to find bh_1, which is 3.53 ft. to right of P_1, we compute, using the values in the table,

$$m = m_1 + (P_1 + P_2 + \ldots + P_{14})3.53$$
$$= 587{,}996 + 46{,}194 \times 3.53 = 751{,}061;$$

$\therefore\ bh_1 = 7.51061$ ft.

This method is little, if any, longer than that involving the use of cross-section paper. In fact, by the use of Crelle's 3-place multiplication table, all the multiplications can be effected so expedi-

QUANTITIES PERTAINING TO ARCH (FIG. 15).

1	2	3	4	5	6	7	8	9
Pt. a.	y.	y^2.	z.	z^2.	b_R.	b_L.	$(b_L-b_R)z$.	$(b_R+b_L)y$.
1, 28	2.88	8.29	39.30	1544.49	7.51	10.60	$3.09z_1=121.44$	$18.11y_1=52.16$
2, 27	6.49	42.12	32.16	1034.27	4.69	6.40	$1.71z_2=54.99$	$11.09y_2=71.97$
3, 26	8.39	70.39	27.35	748.02	3.25	4.28	$1.03z_3=28.17$	$7.53y_3=63.18$
4, 25	9.67	93.51	23.50	552.25	2.33	2.93	$.60z_4=14.10$	$5.26y_4=50.86$
5, 24	10.53	110.88	20.20	408.04	1.70	2.03	$.33z_5=6.67$	$3.73y_5=39.28$
6, 23	11.23	126.11	17.40	302.76	1.23	1.42	$.19z_6=3.31$	$2.65y_6=29.76$
7, 22	11.77	138.53	14.90	222.01	0.90	0.98	$.08z_7=1.19$	$1.88y_7=22.13$
8, 21	12.13	147.14	12.65	160.02	0.64	0.66	$.02z_8=.25$	$1.30y_8=15.77$
9, 20	12.48	155.75	10.52	110.67	0.45	0.45		$.90y_9=11.23$
10, 19	12.73	162.05	8.52	72.59	0.29	0.29		$.58y_{10}=7.38$
11, 18	12.88	165.89	6.53	42.64	0.17	0.17		$.34y_{11}=4.38$
12, 17	12.98	168.48	4.60	21.16	0.09	.09		$.18y_{12}=2.34$
13, 16	13.10	171.61	2.76	7.62	.03	.03		$.06y_{13}=.79$
14, 15	13.12	172.13	0.90	0.81	.01	.01		$.02y_{14}=.26$
	150.38 2	1732.88 2			23.29	30.34	$\Sigma(bh.z)=230.12$	$\Sigma(bh.y)=371.49$
					$\Sigma(bh)=53.63$		$=\Sigma(b_L-b_R)z$	
	300.76 $=\Sigma y$	3465.56 $=\Sigma(y^2)$						

tiously and accurately that the tables above and those which follow are very quickly made out. Without such an aid, or an equivalent slide rule, it is no wonder that graphical methods found favor in the past.

The values of y, z, and $bh=b$ (thus found), with their combinations, are entered in the table on p. 82.

40. Proceeding as in Art. 28, we have

$$\mathrm{AD}=\frac{\Sigma(bh)}{\mathrm{N}}=\frac{53.63}{28}=1.916;$$

$$e=\frac{\Sigma(y)}{\mathrm{N}}=\frac{150.38}{14}=10.74;$$

$$\Sigma(ka\,.\,y)=\Sigma(y^2)-\frac{(\Sigma y)^2}{\mathrm{N}}=3465.56-3230.59$$
$$=235.17;$$

$$\Sigma(mb\,.\,y)=\frac{\Sigma(bh)}{\mathrm{N}}\Sigma(y)-\Sigma(bh\,.\,y)$$
$$=1.916\times300.76-371.49=204.76;$$

$$kc=\frac{\Sigma(ka\,.\,y)}{\Sigma(mb\,.\,y)}mb=\frac{235.17}{204.76}mb=1.148mb;$$

$$\mathrm{H}=\frac{204.76}{235.07}(100{,}000)=87{,}090 \text{ lbs.};$$

$$\frac{\mathrm{V}}{100000}=\frac{v_1m_1}{z_1}=\frac{\Sigma(b_L-b_R)z}{\Sigma(z^2)}$$
$$=\frac{230.12}{10454.7}=0.02201.$$

$\therefore$ V$=$2201 lbs., $\quad v_1m_1=39.3\times.022=0.865.$

Similarly, any $vm=0.022z$.

41. Following the suggestions of Art. 32, the following table for the *right half* is made out. All values are in feet.

Point.	$vm = .022z$.	$mh = 1.92 - vm$.	$mb = mh - bh_R$.	$kc = 1.15mb$.	$e + kc = 10.74 + kc$.	$ac = e + kc - y$.
O′	0.96	0.96	−8.68	−9.98	+0.76	+0.76
a_1	.87	1.05	−6.46	−7.43	3.31	+ .43
a_2	.71	1.21	−3.48	−4.00	6.74	+ .25
a_3	.60	1.32	−1.93	−2.22	8.52	+ .13
a_4	.52	1.40	− .93	−1.07	9.67	+ .00
a_5	.44	1.48	− .22	− .25	10.49	− .04
a_6	.38	1.54	+ .31	+ .35	11.09	− .14
a_7	.33	1.59	+ .69	+ .79	11.53	− .24
a_8	.28	1.64	+1.00	+1.15	11.89	− .24
a_9	.23	1.69	+1.24	+1.42	12.16	− .30
a_{10}	.19	1.73	+1.44	+1.65	12.39	− .34
a_{11}	.14	1.78	+1.61	+1.85	12.59	− .29
a_{12}	.10	1.82	+1.73	+1.99	12.73	− .25
a_{13}	.06	1.86	+1.83	+2.10	12.84	− .24
a_{14}	.02	1.90	+1.89	+2.17	12.91	− .21
Crown	.00	1.92	+1.92	+2.21	12.95	− .17

The values of z, bh, and y are taken from the preceding table. For point O′, $z = 43.55$ ft. and $y = 0$. For crown, $y = 13.12$.

The next table gives the ordinates in feet for the *left half* of the arch. Since the symmetrical points a_1 and a_{28} have the

same z (which is *not* an abscissa, only a distance, always positive), and the same is true for all symmetrical points, the values of $vm = .022z$ can be taken from the preceding table. Although the columns for z, bh, and y are not repeated in the last two tables to save space, it is advisable in practice to repeat them to avoid mistake.

Point.	$mh = 1.92 + vm$.	$mb = mh - bh_L$.	$kc = 1.15mb$.	$e + kc = 10.74 + kc$.	$ac = e + kc - y$.
O	2.88	−10.82	−12.44	− 1.70	−1.70
a_{28}	2.79	− 7.81	− 8.98	+ 1.76	−1.12
a_{27}	2.63	− 3 77	− 4.34	+ 6.40	− .09
a_{26}	2.52	− 1.76	− 2.02	+ 8.72	+ .33
a_{25}	2.44	− .49	− .56	+10.18	+ .51
a_{24}	2.36	+ .33	+ .38	+11.12	+ .59
a_{23}	2.30	+ .88	+ 1.01	+11.75	+ .52
a_{22}	2.25	+ 1.27	+ 1.46	+12.20	+ .43
a_{21}	2.20	+ 1.54	+ 1.77	+12.51	+ .38
a_{20}	2.15	+ 1.70	+ 1.95	+12.69	+ .21
a_{19}	2.11	+ 1.82	+ 2.09	+12.83	+ .10
a_{18}	2.06	+ 1.89	+ 2.17	+12.91	+ .03
a_{17}	2.02	+ 1.93	+ 2.22	+12.96	− .02
a_{16}	1.98	+ 1.95	+ 2.24	+12.98	− .12
a_{15}	1.94	+ 1.93	+ 2.22	+12.96	− .16

The values of t at O, O′, and the crown have both been put in the column headed "*ac*," for convenience. Of course there

is no point a at either O, O′, or the crown, and these values must be excluded in forming the sums

$$\Sigma(ac), \quad \Sigma(ac\,.\,y), \quad \Sigma(ac\,.\,x),$$

which by Art. 19 should be zero if the points c have been located accurately.

42. These sums will rarely be exactly zero on account of writing results only to the nearest hundredth. Thus we took $kc=1.15mb$, whereas it is $kc=1.148mb$. In this example, although the sums are not zero, the ratios of the + and − terms are nearly unity. Thus

$$\frac{\Sigma(+ac)}{\Sigma(-ac)}=\frac{3.91}{3.82}; \quad \frac{\Sigma(+ac\,.\,y)}{\Sigma(-ac\,.\,y)}=\frac{23.80}{21.94};$$

$$\frac{\Sigma(+ac\,.\,x)}{\Sigma(-ac\,.\,x)}=\frac{75.76}{72.11}.$$

If the points c are lowered 0.01 ft., the ratios all become less than unity or the reverse of the given ratios.

It is plain, then, that points c have been established with an error less than 0.01 ft., or practically exactly.

The distances a_1c_1, a_2c_2, . . . , $a_{28}c_{28}$ can now be laid off to scale; above points a when ac is positive, below when negative.

Similarly lay off $t = -1.70$ below O, and $t = +0.76$ above O′, to get points in the true equilibrium polygon at the springs.

The position of the points c can be roughly tested by drawing the equilibrium polygon c in the usual manner, starting at the center of pressure at the crown 0.17 ft. below the center of the joint, and using the true pole P of the force diagram on the left of Fig. 15, where the loads are laid off to scale.* To find P measure downwards to scale of loads from the intersection of ray 14–15 with load line, V=2201 lbs., then horizontally to the right, H=87,090 lbs., to pole P.

43. The bending moment at any point $= M = Ht = 87{,}090ac$ ft.-lbs., the value of ac or t being taken from the last two tables. The value of M can likewise be found from the formula of Art. 34:

$$M = H(y' + t_c) - m \pm Vz.$$

* Neither polygon b nor c can be accurately drawn by the usual construction unless checked and corrected at intervals by computation. It will doubtless prove a wholesome experience to the young draftsman to draw the polygon b with a horizontal thrust at the crown = 100,000 lbs., and *after* drawing it, compare results with the computed ordinates at the P's given above.

If no checks are applied in constructing polygon b, it may be regarded as worse than worthless, because it tends to give security in its false results. In the writer's "Voussoir Arches," he has given other fairly accurate methods of constructing polygon c.

On dividing the M thus found for any point by H, we find the value of *ac* for that point.

It is evident that this method of finding the values of *ac* is much more tedious and liable to error than the method given above.

It will be observed from the last two tables that at O, $t=-1.70$, its greatest value; at a_{28}, $ac=-1.12$. Polygon *c* now crosses the neutral line near a_{27}, passes above it and attains a maximum at a_{24}, where $ac=0.59$. It then gradually approaches the neutral line, crosses it near a_{17}, passes 0.17 below the center of the crown section, attains another maximum at a_{10}. where $ac=-.34$, then approaches the neutral line, crosses it at a_4 and passes 0.76 ft. above it at O′. The equilibrium polygon *c* keeps within the middle third of the arch ring at all points but a_{24}, a_{28}, and O. At a_{24} it passes only 0.02 ft. outside, but at the radial joint at O it passes 0.25 ft. below the middle third limit. Hence if there is to be no tension at the left springing section, the radial depth of section must be increased from 6 to 7.5 ft. This will be considered later.

44. To find the total normal and shearing stresses on the radial section at O, draw through the true pole P a line perpendicular to the radius to the neutral

line at O, and from the end of the lower ray of the force diagram (which, to scale, is the resultant on joint at O), drop a perpendicular (the shearing force N) to the first-named perpendicular, cutting off on it the component T of the resultant parallel or tangential to the neutral line at O. These lines measured to the scale of loads give, at O, T = 118,400, N = 22,800 lbs. Similarly, the resultant (given by the proper ray of the force diagram) on any section can be decomposed into shearing and tangential components. Thus at a_{24} it is found that T = 90,000, N = 0. The bending moment at O = tH = $-1.7 \times 87{,}100 = -148{,}070$ ft.-lbs.; at a_{24} the moment is $+.59\text{H} = +51{,}389$ ft.-lbs.; at a_{10}, $\text{M} = -.34\text{H} = -29{,}614$; at O′, $\text{M} = +.76\text{H} = +66{,}196$ ft.-lbs.

45. The unit stress at the upper or lower edge of any (radial) section of the arch ring is given by the formula

$$s = \frac{\text{T}}{\text{A}} \pm \frac{\text{H}tv}{\text{I}}.$$

For the same section the only quantities that can vary with the position of the live load are T, H, and t. Generally s is a maximum or minimum when $\text{M} = \text{H}t$ is a maximum or minimum. It is only for values of M nearly the same that the influence of T may change these results.

For a given arch it is only possible by treating single loads and combining results to find, with certainty, that position of the live load that gives maximum stresses at a given section. (See Art. 75.) It is found in this way that for usual segmental or three-centered arches the live load should extend past O approximately the following parts of the span OO′ of the neutral line, for maximum stresses at the section given:

Section O, 0.35 to 0.4 span;
Section at a_{24}, 0.35 to 0.4 span;
Section at a_{10}, 0.5 to 0.55 span;
Section at O′, 0.6 to 0.65 span.

In case, then, the method of single loads is not used (which alone can give certainty), the three positions of the live load covering, say, ¾ half-span, half-span, and ⅝ span should be used.

In Fig. 15 the live load covers ¾ half-span nearly, so that the results above should give us the maximum compression at the intrados at section O, and at the extrados at section at a_{24}.

We shall, before taking up the half-span loading, show that on shifting the live load a little to the right, so that its front reaches to the vertical through a_{18}.

the moments at O and a_{24} are materially diminished.

LIVE LOAD EXTENDING UP TO a_{18}

46. The table of moments for the right half is unaltered, also the values of m_{16}, m_{17}, m_{18}, and m_{19} given there, so that the new table for the left half begins as follows:

P	R	a	aR	m
	5,161,			$22,009 = m_{19}$
$P_{19} = 3,128$,	8,289,	2.00,	16,578,	$38,587 = m_{20}$
$P_{20} = 3,259$,	11,548,	2.07,	23,904,	$62,491 = m_{21}$
$P_{21} = 3,363$,	14,911,	2.15,	32,059,	$94,550 = m_{22}$
. .	. .	. .	. .	. .

The values of loads P_{21}, P_{22}, . . . P_{29} are taken from the preceding table, and the table completed as before. The values of b_L are then found, the values of b_R taken from a preceding table, and a table of quantities prepared as before. We thus find $\Sigma(bh) = 57.47$, $\Sigma(b_L - b_R)z = 338.61$, $\Sigma(bh \,.\, y) = 400.43$. The values Σy, $\Sigma(y^2)$, $\Sigma(z^2)$, $e = 10.74$, $\Sigma(ka \,.\, y) = 235.17$ remain unaltered, however the load is shifted. Hence proceeding as in Art. 28, we derive $\Sigma(mb \,.\, y) = 216.93$, $vm = 0.0324z$, $kc = 1.084mb$, H = 90,140 lbs.

We shall only compute t or ac for four points. At O, $t = -1.51$; at a_{24}, $ac = +.52$; at a_{10}, $ac = -.39$, and at O′, $t = +.98$. Thus the bending moments corresponding are as follows:

At O, $M = -1.51 \times 90,100 = -136,000$ ft.-lbs.
a_{24}, $M = + \ .52 \times 90,100 = + \ 46,800$ "
a_{10}, $M = - \ .39 \times 90,100 = - \ 35,100$ "
O′, $M = + \ .98 \times 90,100 = + \ 88,300$ "

By comparing these values with those given before, we see that the shifting of the load has

materially decreased the moment at O, decreased it at a_{24}, and materially increased the moments at a_{10} and O′.

LIVE LOAD FROM LEFT ABUTMENT TO CROWN.

47. For the live load covering the half-span, the computation will be given in full.

The table of moments for the right half remains the same as in the first case; hence the values of b_R are taken from the first table of quantities.

The moments from which the values of b_L are found are given in the adjoining table.

P (lbs.)	R.	a.	aR.	m (foot-pounds).
P_{15} = 1436	1436	1.39	1996	m_{16} = 1996
P_{16} = 2770	4206	1.86	7823	m_{17} = 9819
P_{17} = 2907	7113	1.89	13444	m_{18} = 23263
P_{18} = 3012	10125	1.98	20047	m_{19} = 43310
P_{19} = 3128	13253	2.00	26506	m_{20} = 69816
P_{20} = 3259	16512	2.07	34180	m_{21} = 103996
P_{21} = 3363	19875	2.15	42731	m_{22} = 146727
P_{22} = 3747	23622	2.43	57401	m_{23} = 204128
P_{23} = 4337	27959	2.62	73253	m_{24} = 277381
P_{24} = 5070	33029	3.05	100738	m_{25} = 378119
P_{25} = 6074	39103	3.60	140771	m_{26} = 518890
P_{26} = 7789	46892	4.30	201636	m_{27} = 720526
P_{27} = 10569	57461	6.00	344766	m_{28} = 1065292
P_{28} = 18309	75770	6.36	481897	m_{29} = 1547189
P_{29} = 17955	93725	1.38	129340	O, m = 1676529
93725			1676529	

The last moment was tested by taking moments of all the P's about O. This checks the entire table. On dividing any m by $H' = 100{,}000$, the assumed horizontal thrust, we find the ordinates to the equilibrium polygon b at the P's. It is sufficiently near to take the mean of any two adjacent to get the following values of $bh = b$:

$$b_{15} = .01,\ b_{16} = .06,\ b_{17} = .16,\ b_{18} = .33,$$

$$b_{19} = .56,\ b_{20} = .87,\ b_{21} = 1.25,\ b_{22} = 1.76.$$

The remaining values were computed (Art. 39):

$$b_{23} = 2.38,\ b_{24} = 3.24,\ b_{25} = 4.42,$$

$$b_{26} = 6.09,\ b_{27} = 8.60,\ b_{28} = 13.33;$$

at O the ordinate to polygon = 16.76.

In the table on p. 94 the values of b_R and b_L are entered, and the sums and products indicated, made out.

Using the formulas of Art. 28, we have

$$\mathrm{AD} = \frac{\Sigma(bh)}{\mathrm{N}} = \frac{66.35}{28} = 2.37;$$

$$\Sigma(mb.y) = 2.37 \times 300.76 - 475.11 = 237.69;$$

$$\frac{\mathrm{V}}{100000}=\frac{vm}{z}=\frac{\Sigma(b_L-b_R)z}{\Sigma(z^2)}=\frac{561.06}{10455}=.05366.$$

$$\therefore\ \mathrm{V}=5366\ \text{lbs.};\quad vm=0.0537z.$$

$$kc=\frac{\Sigma(ka\,.\,y)}{\Sigma(mb\,.\,y)}mb=\frac{235.17}{237.69}mb=0.9894mb\,;$$

$$\mathrm{H}=\frac{237.69}{235.17}(100{,}000)=101{,}000\ \text{lbs.}$$

LIVE LOAD FROM LEFT ABUTMENT TO CROWN.

Point a.		b_R.	b_L.	$(b_L-b_R)z$.	$(b_R+b_L)y$.
1,	28	7.51	13.33	228.72	60.02
2,	27	4.69	8.60	125.75	86.25
3,	26	3.25	6.09	77.67	78.36
4,	25	2.33	4.42	49.11	65.27
5,	24	1.70	3.24	31.11	52.02
6,	23	1.23	2.38	20.01	40.54
7,	22	.90	1.76	12.81	31.31
8,	21	.64	1.25	7.72	22.92
9,	20	.45	.87	4.42	16.47
10,	19	.29	.56	2.30	10.82
11,	18	.17	.33	1.04	6.44
12,	17	.09	.16	.32	3.25
13,	16	.03	.06	.08	1.18
14,	15	.01	.01	.00	.26
		23.29	43.06	561.06	475.11
		$\Sigma(bh)=66.35$		$=\Sigma(bh.z)$	$=\Sigma(bh\,.\,y)$

Hence, confining our attention to points O, a_{24}, crown, a_{10}, and O′, we have:

Point.	$vm = .0537z.$	$mh = 2.37 \pm vm.$	$bh.$	$mb = mh - bh.$	$kc = .989mb.$	$10.74 + kc - y = ac = t.$
O	2.34	4.71	16.76	−12.05	−11.92	−1.18
a_{24}	1.08	3.45	3.24	+ .21	+ .21	+ .42
Crown	.00	2.37	.00	+ 2.37	+ 2.34	− .04
a_{10}	.46	1.91	.29	+ 1.62	+ 1.60	− .39
O′	2.34	.03	9.64	− 9.61	− 9.50	+1.24

M at O $= -119{,}180$ ft.-lbs.

" " $a_{24} = +\ 42{,}420$ "

" " $a_{10} = -\ 39{,}390$ "

" " $O' = +125{,}240$ "

From these results it will be perceived that the bending moments at O and a_{24} have still further decreased, and at a_{10} and O′ increased.

This position of the load gives (about) the maximum stress at a_{10}. For O′ the live load will be extended up to the vertical through a_9. This point is $\frac{3}{4}$ half-span from O′.

LIVE LOAD FROM LEFT ABUTMENT TO a_9.

48. The moments m for the left half are now the same as in the preceding case, with the resulting values of b_L. Likewise $m_{13}=m_{16}$ of preceding table, $m_{12}=m_{17}$, $m_{11}=m_{18}$, $m_{10}=m_{19}$, $m_9=m_{20}$, and $m_8=m_{21}=103,996$.

The remaining moments for the right half are given in the adjoining table.

LIVE LOAD FROM LEFT ABUTMENT TO a_9.

P (lbs.).	R.	a.	aR.	m (foot-pounds).
16512	16512		103996	$m_8=$ 103996
$P_8=$ 1803	18315	2.15	39377	$m_7=$ 143373
$P_7=$ 2060	20375	2.43	49511	$m_6=$ 192884
$P_6=$ 2440	22815	2.62	59775	$m_5=$ 252659
$P_5=$ 2940	25755	3.05	78553	$m_4=$ 331212
$P_4=$ 3652	29407	3.60	105865	$m_3=$ 437077
$P_3=$ 4887	34294	4.30	147464	$m_2=$ 584541
$P_2=$ 6784	41078	6.00	246468	$m_1=$ 831009
$P_1=$12924	54002	6.36	343453	$m_0=$1174462
$P_0=$13680	67682	1.38	93401	O′, $m=$1267863
67682			1267863	

Assuming, as before, a trial thrust of 100,000 lbs., the ordinates b_R are found and entered with those for b_L in the next

table. The computations then proceed as before.

Point a.	b_R.	b_L.	$(b_L-b_R)z$.	$(b_R+b_L)y$.
1, 28	10.20	13.33	123.01	67.77
2, 27	6.83	8.60	56.92	100.14
3, 26	5.03	6.09	28.99	93.30
4, 25	3.80	4.42	14.57	79.49
5, 24	2.92	3.24	6.46	64.86
6, 23	2.21	2.38	2.96	51.55
7, 22	1.76	1.76	0.00	41.33
8, 21	1.25	1.25		30.32
9, 20	.87	.87		21.72
10, 19	.56	.56		14.26
11, 18	.33	.33		8.50
12, 17	.16	.16		4.15
13, 16	.06	.06		1.57
14, 15	.01	.01		.26
	35.99	43.06	232.91	579.32
	$\Sigma(bh)=79.05$		$\Sigma(b_L-b_R)z$	$\Sigma(bh.y)$

Proceeding exactly as in previous cases, we find

$$AD=2.823, \qquad vm=0.0223z,$$

$$kc=0.872mb, \qquad V=2230 \text{ lbs.},$$

$$H=114{,}700 \text{ lbs.},$$

whence the following values of t or ac were computed.

Point.	$vm = .0223z$.	$mh = 2.82 \pm vm$.	bh.	$mb = mh - bh$.	$kc = .872mb$.	t or $ac = 10.74 + kc - y$.
O	.97	3.79	16.76	−12.97	−11.31	−0.57
a_{24}	.45	3.27	3.24	+ .03	+ 0.03	+0.24
a_{10}	.19	2.63	.56	+ 2.07	+ 1.80	−0.19
O′	.97	1.85	12.68	−10.83	− 9.44	+1.30

On multiplying the values of t by H= 114,700, we find the following bending moments M:

M at O = 65,400 ft.-lbs.
" " a_{24} = 27,600 "
" " a_{10} = 21,800 "
" " O′ = 149,110 "

The moments have all diminished except the last, which is very nearly the maximum.

49. The following summary of moments for the different cases is very instructive:

Load up to	a_{20}	a_{18}	crown	a_9
M at O	−148,100	−136,050	−119,180	− 65,380
" " a_{24}	+ 51,389	+ 46,850	+ 42,400	+ 27,530
" " a_{10}	− 29,614	− 35,140	− 39,400	− 21,800
" " O′	+ 66,196	+ 88,300	+125,200	+149,110

It is seen in this example that the maximum negative moment at O is about equal to the maximum positive moment at O′. In this arch, for dead load only, $t = -0.27$ at O or O′ (Art. 51), and $H = 72{,}380$ lbs.; $\therefore$ $M = 19{,}540$ ft.-lbs. Hence for a similar design, where t for dead load is small and negative at O or O′, it would seem that if an engineer is pressed for time in designing an arch and only one position of the live load can be tried, it should cover $\frac{3}{8}$ span or $\frac{3}{4}$ half-span. The maximum moments at O and a_{24} will be given (nearly) by this position, and the positive maximum moment at O′ can be assumed equal numerically to the negative moment at O.

As to a_{10}, increase the negative moment there by $\frac{1}{3}$ or more to get roughly its maximum moment for the load covering the half-span.

50. A word may be added as to finding thrusts and shears at a_{10} and O′. For a_{10}, when the live load covers only the left half of the arch, the load line above ray 14–15 is unaltered; hence from where this ray meets the load line, lay off $V = 5366$ lbs., to scale of loads, downwards to a point; from thence draw horizontally to the right $H = 101{,}030$ lbs., to same scale, to P, the true pole for this position of the load. Through P draw a

perpendicular to the radial section at a_{10}. The total thrust on this section is given by ray 9–10 from pole P. The component of this along this perpendicular is found to be T = 101,800, and the one normal to the perpendicular, the shear, N = 1600 lbs.

For O′ the live load extends to a_9 and the portion of it to the right of the crown = 8000 lbs. Extend the load line upwards to Q (not shown in Fig. 15) a distance = 8000 lbs. to scale; lay off V = 2228 lbs., H = 114,700 lbs., as usual, to find the new pole P. Then PQ to scale is the resultant thrust at O′. On decomposing it into components normal and parallel to the radial section at O′, we find T = 134,000 lbs., N = 12,400 lbs.

The term radial section here and elsewhere refers to the neutral line radius along which the section is drawn.

ARCH SUBJECTED ONLY TO ITS OWN WEIGHT.

51. The loading being symmetrical as to the crown, the sums below will be taken in this case to refer only to the half-arch, and the results doubled when the whole arch is considered. The values of y and $b\kappa$ are taken from the table of quantities (Art. 39):

$$AD=\frac{2\Sigma(b_R)}{N}=\frac{23.29}{14}=1.664;$$

$$\Sigma(mb\,.\,y)=\frac{2\Sigma(b_R)}{N}\Sigma y-2\Sigma(b_R\,.\,y).$$

Here, to find $\Sigma(b_R\,.\,y)$, each b_R as given in the table must be multiplied by the y corresponding and the sum taken. Thus

$$\Sigma(mb\,.\,y)=1.664\times300.76-330.24$$
$$=170.22.$$

From Art. 40, $\Sigma(ka\,.\,y)=235.17$.

$$\therefore\ kc=\frac{235.17}{170.22}mb=1.382mb;$$

$$H=\frac{170.22}{235.17}(100{,}000)=72{,}380 \text{ lbs.}$$

Since for any symmetrical load the closing line mm_1 is horizontal, vm is zero. A few of the computations are given to show the method of forming the table.

Point.	bh.	$mb=$ $1.66-bh$.	$kc=$ $1.38mb$.	$e+kc=$ $10.74+kc$.	y.	$ac=$ $e+kc-y$.
O′	9.64	−7.98	−11.01	−0.27	0	−0.27
a_1	7.51	−5.85	− 8.07	+2.67	2.88	−0.21

The values obtained for t or ac are as follows:

At
O′, $t=-0.27$; $a_1c_1=-0.21$; $a_2c_2=+.07$; $a_3c_3=+.16$; $a_4c_4=+.15$; $a_5c_5=+.15$; $a_6c_6=+.10$; $a_7c_7=+.02$; $a_8c_8=+.02$; $a_9c_9=-.07$; $a_{10}c_{10}=-.10$; $a_{11}c_{11}=-.08$; $a_{12}c_{12}=-.07$; $a_{13}c_{13}=-.11$; $a_{14}c_{14}=-.10$;
at crown, $t=-.09$.

Note that the equilibrium polygon passes *below* O′ and a_1.

When the arch is subjected to its own weight and to a rolling load covering the whole span besides, use the values of b_L from the table, Art. 47, for the left half loaded, and proceed exactly as above. It is found that H = 129,700 lbs. and that it acts 0.01 ft. below the center of the crown section, so that the moment there is $129{,}700\times(-.01)=-1300$ ft.-lbs.

The resultant at the springing section now acts *above* O a distance $t=+0.19$ ft. It has the largest horizontal thrust thus far found, and it should be combined with the weight of the abutment acting along the vertical through its center of gravity, to find the center of pressure on the base. The resultant at O′, when the load extends to a_9 or covers $\frac{5}{8}$ of the

span, should likewise be used, since it acts the greatest distance above O′.

52. The thrust and moment at the crown can also be found very briefly as follows:

The moment at the crown, Art. 47, due to live load on one half span and dead load is $101{,}030 \times -.04 = -4041.20$ ft.-lbs. The moment due to dead load only is $72{,}380 \times -.09 = -6514$ ft.-lbs. The difference $+2473$ is the moment due to live load only, and $2 \times 2473 = +4946$ is the moment for the live load covering the whole span, since the thrust, which is doubled when the right half also is loaded, acts at the same point at the crown as before.

The horizontal thrust at the crown due to live load only, over the whole span, is

$$2(101{,}030 - 72{,}380) = 57{,}300 \text{ lbs.}$$

On adding the thrust due to dead load to this, we have

$$\mathrm{H} = 129{,}680.$$

The total moment at the crown due to dead and live loads being

$$\mathrm{M} = +4946 - 6514 = -1568 \text{ ft.-lbs.},$$

the thrust acts below the center of the crown section:

$$t = \frac{-1568}{129{,}680} = -0.012 \text{ ft.},$$

as found above another way. The value of H agrees with that first found, and the value of M differs from the former value because t was carried only to hundredths in the first case.

REVISION OF SECTION. REINFORCEMENT.

53. It was seen in Art. 43 that the section at O (and hence at O′) must have a radial depth of 7.5 feet. To effect this, measure along the radius to the neutral line at O, 3.75 ft. above and below O to define points in the extrados and intrados. Pass a circular curve through the latter, tangent to the intrados near a_{27}, say. The radius measured was $r=30.4$. A similar curve is drawn at the right spring, so that the new clear span is $90-2.50=87.50$ ft. Through the point established on the extrados on section at O, draw a tangent to old extrados to give a new extrados for portion considered. These changes are shown by dotted lines on the figure.

In the notation of Art. 37, Fig. 13, we have now approximately, by scaling only, $a=31.3$, $h=7.05$, $R=72.97$, $r=30.4$, to define the revised intrados, which is a new three-centered curve, giving sufficient depth at a_1, a_{28}, O, and O′.

The centers of pressure on the sections at O, a_{20}, and a_{24} are practically at the middle third limits, so that the arch ring has been economically designed.

54. To meet temperature and allied stresses, however, the concrete must be reinforced with steel.

Mr. Edwin Thacher, M. Am. Soc. C. E., has suggested * "that the steel must be capable within its elastic limit of taking the entire bending moment of the arch without aid from the concrete and have a flange area of not less than 1/150 part of the total area at the crown."

He pertinently adds: "Though the steel can never take the entire bending moment under normal conditions, it is a comfort to an engineer to be assured that if, through the carelessness or dishonesty of a contractor, or the incompetency or connivance of an inspector, any part of the concrete should be of poor quality, or the specifications as to continuous work be disregarded, his work will still continue to stand and do its duty."

The suggestion affords a basis for estimating the sectional area A_2 of the steel for the width, 1 foot, of the slice of the arch considered.

Half of the steel will be supposed placed 3 inches from the extrados and half 3 inches from the intrados, so that at section O the bars will be 7 ft. apart. If the elastic limit of the steel is 36,000 lbs. per sq. in., and A_2 be ex-

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* *Engineering News*, Sept. 21, 1899, p. 179.

pressed in square feet, the resisting moment in foot-pounds is

$$\frac{A_2}{2}\times 7\times 36{,}000\times 144.$$

From a preliminary estimate (to be given later), the bending moment at O, due to a change of temperature of 15° F., is about 100,000 ft.-lbs.; to this add the maximum moment due to dead and live loads, say 150,000 ft.-lbs., and we have a total bending moment at O of 250,000 ft.-lbs.

On equating this with the preceding, we derive the net section $A_2=0.014$ sq. ft. This is less than 1/150 area crown, or $\frac{2}{3}$ of 1% of $2.75=0.018$. We shall assume $A_2=0.016$ sq. ft., as it seems ample to meet the demands.

APPROXIMATE STRESSES.

55. The division of the neutral axis used in Fig. 15 pertains strictly to a plain concrete or masonry arch. Many constructors in this country assume this division to be sufficiently correct for a reinforced arch. The true division for a reinforced arch will be given in Art. 63. It necessarily involves more time in com-

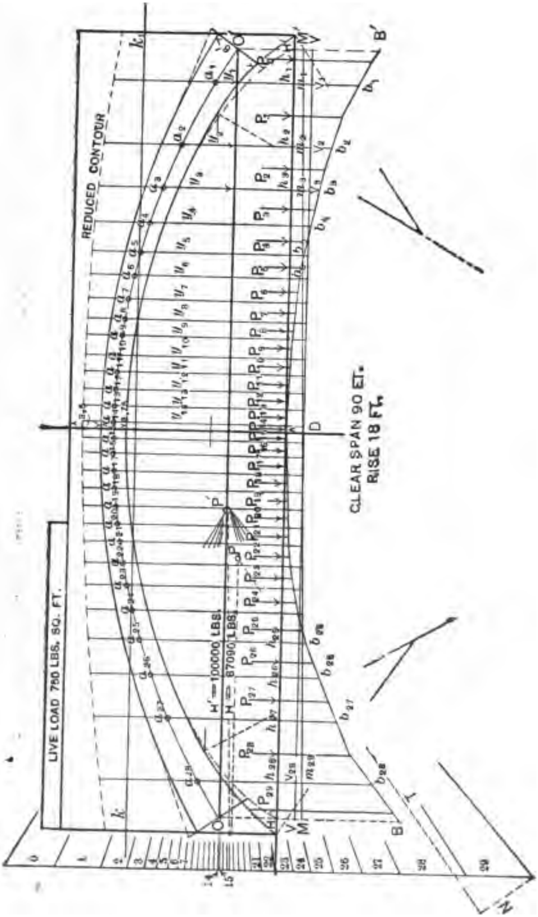
LIVE LOAD 750 LBS. SQ. FT.
REDUCED CONTOUR
H' = 100000 LBS.
H = 87090 LBS.
CLEAR SPAN 90 FT.
RISE 18 FT.

FIG. 15.

putation than for a concrete arch, but when the "diagram" is once (even partially) drawn, the labor is the same in either case.

Assuming, then, for the present that the maximum bending moments and thrusts found above are approximately correct for the same arch reinforced and revised in section, we proceed to find the stresses at the critical sections by formulas 8 of Art. 16.

Taking the modulus of elasticity of concrete as 2,000,000, and that of steel as 30,000,000 lbs. per sq. in., it follows that $n=15$. Hence $nA_2=15\times0.016=0.24$, and $d_2=d_1-0.5$. The formulas now reduce to

$$s_1=\frac{T}{d_1+0.24}\pm\frac{Md_1}{\frac{1}{6}d_1^3+0.12d_2^2};$$

$$s_2=15\left\{\frac{T}{d_1+0.24}\pm\frac{Md_2}{\frac{1}{6}d_1^3+0.12d_2^2}\right\}.$$

These stresses are in pounds per square foot and will be reduced to pounds per square inch, to insert in table, in which T = thrust normal to section, M the maximum bending moment, and d_1 the radial depth.

Point.	T. Lbs.	Shear. Lbs.	M. Ft.-Lbs.	d_1. Ft.	Stress Intrados. Lbs. per Sq. In.	Stress Extrados. Lbs. per Sq. In.
O	118400	22800	−148070	7.50	s_1= 208 s_2= 3010	s_1= 5 s_2= 177
a_{28}	108300	10140	− 97500	5.40	s_1= 260 s_2= 3710	s_1= 8 s_2= 292
a_{24}	90000	0	+ 51389	3.22	s_1= 12 s_2= 577	s_1= 350 s_2= 4860
a_{10}	101800	1600	− 39390	2.81	s_1= 410 s_2= 5660	s_1= 55 s_2= 1300
O′	134000	12400	+149110	7.50	s_1= 19 s_2= 396	s_1= 223 s_2= 3230

s_1= *stress in pounds per square inch in concrete;*
s_2= *stress in pounds per square inch in steel.*

It will be observed from the results in the table that the largest stress in the concrete is 410 lbs. per sq. in. at a_{10}, and that there is no tension anywhere. The maximum stress in pounds per square inch in the steel is only 5660.

CHAPTER IV.

TEMPERATURE AND ALLIED STRESSES.

56. The stresses due to temperature for the arch without hinges are very large when the range of temperature t° above and below the *mean* is large. It will be supposed that the arch is without weight and that it exactly fits between the skewbacks without stress at this mean temperature. In other words, if the arch was laid on its side on a perfectly smooth horizontal platform, with the assumed span, skewbacks, and rise, it would be without stress at the mean temperature.

This range t°, which will be expressed in Fahrenheit degrees, is fully equal or greater than the range of the atmosphere for metal arches, but it is nothing like so great for stone or concrete arches, otherwise we should be forced to stop building them. The stone, earth filling, and concrete are such great non-conductors of heat that the average range for the whole arch above and below the mean doubtless does not exceed 20° F., or 40° F.

entire change. The main reason for this belief is the fact that stone and concrete arches have stood for so many thousand years, whereas if they had actually experienced large extremes of temperature, they would have crumbled long ago. One direct observation can likewise be appealed to. Professor Howe in "Symmetrical Masonry Arches," p. 119, says: "A recording thermometer placed in the ring of a reinforced-concrete bridge having earth filling indicated that the total range of temperature change did not exceed about 20° F. in some ten or twelve months." In open-work spandrels, particularly when the arch is composed of separate ribs of small section, the range must be necessarily greater. The chances are against such thin ribs standing if temperature stresses have been ignored in their design. They doubtless never experience the range of temperature of steel which is quickly heated to the temperature of the air at its maximum or minimum during the day; whereas it may take days for even a small mass of concrete to attain the average temperature during the interval.

Constructors who utterly ignore temperature stresses in open spandrel work are doubtless laying up for themselves some failures.

In applying the theory that follows to concrete arches, an approximation has to be resorted to: that the arch ring throughout has the same temperature. Of course at extreme air temperatures, the outside of the ring has a different temperature from the inside or the part next the earth filling. The theory, however, exactly applies to those periods when the temperature is uniform throughout.

The coefficient of expansion per degree Fahrenheit, ε, has been given values varying from .0000055 for a 1:2:4 concrete, to .0000065 for a 1:3:6 concrete. Although even higher values have been given, in what follows ε will be assumed to equal .000006.

57. If l is the span of the neutral line in feet, and we suppose the temperature to rise t° F. above the mean, if the ends of the arch were free the span would be increased $l\varepsilon t^\circ$ feet, since the increase is made up of the horizontal projections of the changes for each small portion of the arch.

To bring these ends back to the first position, it will suffice to suppose horizontal forces acting inwards at each spring. The arch will now become more bowed and the end tangents will rise. Since for the arch fixed at the ends the

end tangents must not change position, couples must now be added at each abutment to force the end tangents to assume the original position at the mean temperature. The couple at the left spring must evidently be right-handed to cause a lowering of the end tangent there. For a fall of temperature the left couple will be left-handed, and the forces at the springs will act outwards.

In Fig. 16 let aa_8 represent part of the neutral line and ab the half-span. To fix the ideas, suppose a rise of temperature $t°$; then there will be a horizontal reaction H at a, acting from a towards b, and a right-handed couple also at a. From symmetry at the right spring there will be a reaction H acting inwards, and a left-handed couple of equal moment to that of the couple at a. Let this bending moment $=$ H . ak; then we can suppose it produced by a horizontal force H acting to the right along kk_8 and another horizontal force H acting to the left at a. The latter balances the reaction H there; hence a single force H acting to the right along kk_8 produces the same effect as the reaction and couple supposed at the left springing. A similar state of affairs exists at the right abutment, only H there acts along kk_8 to the left and thus exactly balances the H acting to the

right. The reactions thus exactly balance, which should be the case, since there are no loads horizontal or vertical acting on the arch.

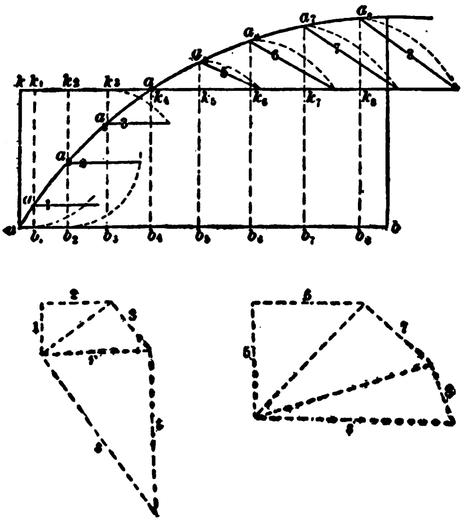

Fig. 16.

Considering again the left half, the bending moment at any point a_6 is H . k_6a_6, due simply to the single force H acting to the right along kk_8. We reach this same result by using the reaction H and

couple with moment H . ak, both acting at the left spring. Thus

$$\mathrm{H} \,.\, b_6a_6 - \mathrm{H} \,.\, ak = \mathrm{H} \,.\, k_6a_6.$$

The moments for points either side of kk_8 will evidently be of different sign.

For a fall of temperature the forces H above are all reversed in direction; otherwise the investigation is similar to the above.

Suppose now the neutral line is divided into such lengths s_1, s_2, . . . that $s \div \mathrm{I}$ for a concrete, or $s \div (\mathrm{I}_1 + n\mathrm{I}_2)$ for a steel-concrete arch, is constant; then to fix the line k_1k_8, we have, by equations 13 of Art. 19, since for any a, $\mathrm{M} = \mathrm{H} \,.\, ka = \mathrm{H}t$,

$$\Sigma(ka) = 0, \quad \Sigma(ka \,.\, x) = 0.$$

These conditions, being the same as those fixing the closing line kk_1 of Art. 25, show that kk_8 lies at a distance $e = (\Sigma y) \div$ N above the line ab, and coincides with the line thus previously established.

Because the reactions considered have produced a virtual change of span $l\epsilon t^\circ$, since the arch altered by temperature has been brought back by the reactions to the original span, we have, by Art. 17,

$$l\epsilon t^\circ = \Sigma \frac{\mathrm{M}ys}{\mathrm{E}_1(\mathrm{I}_1 + n\mathrm{I}_2)} = \mathrm{H}\frac{s}{\mathrm{E}_1(\mathrm{I}_1 + n\mathrm{I}_2)}\Sigma(ka \,.\, y),$$

the summation extending over the entire span. For a non-reinforced arch of any material, make $I_2=0$. Since $ka=y-e$,

$$\Sigma(ka\,.\,y)=\Sigma y^2-e\Sigma y=\Sigma y^2-\frac{(\Sigma y)^2}{N}.$$

$$\therefore\ H=\frac{E\,l\varepsilon t^\circ}{\Sigma y^2-e\Sigma y}\ \frac{I_1+nI_2}{s}.$$

If the dimensions are all in feet E_1 must be expressed in pounds *per square foot;* then H will be given in pounds. If E_1 is in tons per square foot, H will be given in tons.

58. The following construction for $\Sigma(ka\,.\,y)$ is interesting and may serve as a check.

1. For the part below kk_8, describe circles on k_1b_1, k_2b_2, . . . as diameters; the intersections of the horizontals through a_1, a_2, . . . with these give the lines 1, 2, . . . which, measured to the scale of the arch and squared, give the products $\overline{a_1k_1}\cdot\overline{b_1a_1}$, $\overline{a_2k_2}\cdot\overline{b_2a_2}$, etc. Therefore in the figure just below this part of the arch, draw 2 perpendicular to 1 of the lengths above; the square of the hypotenuse of the right triangle is equal to 1^2+2^2. Next draw 3 perpendicular to this hypotenuse; we have then $r^2=1^2+2^2+3^2$.

2. Next consider the part of the arch above kk_8. On $\overline{b_5a_5}$, $\overline{b\,a}$, . . . , as diameters, describe circles; from their intersections with kk_8, draw lines to a_5, a_6, . . . , and denote the lengths of these lines by 5, 6 . . . ; then as before find $s^2=$ the sum of their squares.

3. Draw a perpendicular t to r of such length that s is the hypotenuse of the right tri-

angle whose three sides are r, s, and t. Then $t^2=s^2-r^2=\Sigma\overline{ak}\,.\,y$. Hence measure t to the scale of the arch and square the number found, which square is thus equal to $\Sigma\overline{ak}\,.\,y$ required, whence H may be found from the preceding equation.

Example.—Let the span of a steel arch of constant section be 518 ft., the rise 51.8 ft., EI $=39,680,000$ foot-tons, and $l\epsilon t=0.2735$ feet (ϵ is taken as .000012 for 1° C.), t being 44° C. (about 80° F).

On dividing the semi-arch, drawn to a scale of 20 ft. to the inch, into sixteen equal parts, and proceeding as above, we find $t^2=3944=\Sigma(ka\,.\,y)$; $\therefore$ H$=82.4$ tons. Winkler's formula (omitting the very small term k) gives H$=81.9$ tons. Here $\alpha=22°\ 37'$, $r=673.4$ ft.

See the writer's "Theory of Steel-concrete Arches and Vaulted Structures," p. 72, for Winkler's formula and a comparison between results as given by the correct analytical method and two approximate methods, including the one given above.

59. After H has been computed by the formula above, to find the stresses at any section a, H must be resolved into components perpendicular and parallel to the section. The first-named component $=$T, and since M$=$H . ka, on substituting numerical values for T and M in formulas 8 of Art. 16, the stresses are found.

The following neat construction applies in finding the components when aa_8 is a circular arc. Lay off to scale a horizontal line AB$=$H (not drawn in the figure). Describe a semicircle on AB as a

diameter. Through A draw lines parallel to the radii at a_1, a_2, . . . , to intersection with semicircle. The lines from such intersections to B, to scale, give the values of T at a_1, a_2, . . . , since they are perpendicular respectively to the lines first drawn.

To ascertain the character of the stresses at a section, as that at a_1, conceive the right half of the arch removed and its action upon the left half to be replaced by a single force H acting along kk_8 to the left for a rise, and to the right for a fall, of temperature. Then conceiving two opposed horizontal forces at a_1 each equal to H, the result is a couple whose moment is H . k_1a_1 and a single force at a_1 which acts to the left for a rise, and to the right for a fall, of temperature, as does also its component T.

For a *rise*, T thus causes uniform compression on the cross-section at a_1 and the couple, being left-handed, causes compression at the extrados, tension at the intrados. The greatest unit stress is now compressive at the extrados. Similarly for a *fall*, T induces a uniform tension over the cross-section at a_1 and the couple, being right-handed, now causes tension at the extrados, compression at the intrados, so that finally the extrados suffers the greatest stress, and it is tensile.

It is easily seen that the results are reversed for sections at points a_5, a_6, . . . , which lie above kk_8.

For points in the right half of the arch, conceive the left half removed and that its action on the right half is the same as that of the single force H acting along kk_8 to the right for a rise, and to the left for a fall, of temperature. The conclusions are as above as to the character of the greatest stresses at the edges.

60. As an illustrative example, let us consider the reinforced arch of Arts. 53, 54, accepting the division of the neutral axis as leading to approximately correct results. In this division, $I \div s = [1/12](4.84)^3 \div 10.2 = 0.9263$, $t^\circ = \pm 15^\circ$ F., $l = 90$,* $\varepsilon = .000006$; $\therefore$ $l\varepsilon t = 0.0081$ ft. change of span. Take $E_1 = 2{,}000{,}000 \times 144$ lbs. per sq. ft., $\Sigma y^2 - e\Sigma y = 235.6$.

$$H = \frac{E_1 l \varepsilon t^\circ}{\Sigma y^2 - e\Sigma y} \frac{I}{s} = 9170 \text{ lbs.}$$

As the stresses in the concrete at the extrados and intrados are alone of interest, the following numerical values are inserted in the formula for s_1, Art. 55, and the results divided by 144 to reduce to pounds per square inch.

At O, $T = 7400$, $M = 9170 \times 10.74$, $d_1 = 7.5$.

The results of the substitution and reduction are $+74$, -60, which, being interpreted as above, give:

For a rise, stress = 74 lbs. per sq. in. compression at extrados, 60 tension at intrados.

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* $l = OO' = 87$, but as ε was taken rather small, the round number 90 for l was used.

For a fall of 15° F., stress = 74 tension at extrados, 60 compression at intrados.

At a_{28}, T = 7900, M=9170×7.86, d_1=5.4.

Rise: 93 comp. at ext., 73 tension at int.

Fall: 93 tension at ext., 73 comp. at intrados.

The moment at a_{24} being small, the stress there will not be considered.

At a_{10}, T=9100, M=9170×2, d_1=2.81.

∴ for a rise, the stress is 102 comp. at int., 63 tension at ext.

For a fall, 102 tension at int., 63 comp. at extrados.

These stresses will be combined with other stresses in Art. 62.

CHANGE OF SPAN. ELASTIC SHORTENING OF THE ARCH.

61. In case the abutments give a small amount, the span of the neutral line is increased a small amount Δ. The corresponding value of H is found by replacing $l\epsilon t°$ in the formula above by Δ. The effect is equivalent to a fall of temperature which requires the unstrained arch to be fitted to a larger span.

The result is the same for the elastic shortening of the arch, due to the tangential forces T, causing uniform compression on each cross-section, and hitherto neglected. For a given loading, the sum of the horizontal projections of the shortening of each part of the neutral

line, due to the corresponding T, which can be computed, is the change of span, and replaces ϵt° above.

It is rarely necessary to enter into such refinements, however, simple as the work is, since strictly several loadings would have to be considered.

If the stress f in pounds per square foot, due to T only, was the same on each cross-section of the arch, the change of the span of the neutral line would be $\frac{fl}{E}$, where l is the span in feet and E is the modulus in pounds per square foot (not inch). Howe, in "Symmetrical Masonry Arches," states that "For fixed arches having a ratio of rise to span of 1/10, the effect of the axial stress is to reduce the magnitude of the horizontal thrust about 30%, while for a ratio of 2/10, this percentage drops to about 10%." These percentages must evidently vary very much with the span, dimensions of arch ring, and loading.

For the arch, Fig. 15, when the live load extends from the left abutment to a_{20}, the forces T, divided by the depths (areas) at the points named, give the normal stresses on the sections as follows: At O, 15,300; crown, 34,000; a_{24}, 26,100; a_{10}, 32,900 lbs. per sq. ft.

Take $f=25{,}000$ as a rough average,

then the change of span due to the uniform compression is

$$\frac{fl}{E}=\frac{25{,}000\times 90}{2{,}000{,}000\times 144}=0.008 \text{ ft.},$$

or the same as for 15° F. fall of temperature. The stresses at O, a_{28}, and a_{10} will then be the same and of the same character as for a 15° fall of temperature already computed. As these stresses are caused by the forces T acting along the neutral axis of the arch, let us designate them as "axial" stresses.

62. On combining the maximum stresses previously found in the concrete for the revised reinforced arch of 87.5 ft. clear span, 18 ft. rise, we deduce the following resultant stresses in pounds per square inch. In the table, + stands for compression, − for tension.

Point.	Fiber.	D. & L. Loads.	Axial.	D. & L. & Axial.	Temp. ± 15 F°.	Max. Comp.	Max. Tension.
O	upper	+ 5	− 74	− 69	± 74		−143
	lower	+208	+ 60	+268	± 60	+328	
a_{28}	upper	+ 8	− 93	− 85	± 93		−178
	lower	+260	+ 73	+333	± 73	+406	
a_{10}	upper	+ 55	+ 63	+118	±102		
	lower	+410	−102	+308	± 63	+371	

The steel is subjected to very little stress. Thus at a_{28} the maximum compression at the lower bar, due to all causes, is only 5740, and the tension at the upper bar is only 2330 pounds per square inch.

The steel has thus a great excess of strength, and it would appear at first sight that it should be reduced in section. This would be extremely inadvisable, for no allowance has been made thus far for a slight yielding of the abutments or the unknown shrinkage of the concrete from setting in air. The stresses from these two causes are unfortunately of the same character as those due to a fall of temperature and the axial forces; they thus all conspire together.*

The provision of Mr. Thacher in his specifications, that the steel should be designed to take all the bending moment, is a wise and prudent one. The bending

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* Practically no information exists as to the shrinkage of concrete. The coefficient of shrinkage of 1 : 3 or 1 : 5 cement mortar has been given as varying from 0.0008 to 0.0015. Considère found for a 1 : 3 mortar with a reinforcement of $5\frac{1}{2}\%$ of steel, a coefficient of 0.0001. This coefficient would entail a shortening of the span $= 90 \times .0001 = .009$ ft., or greater than the change due to 15° fall of temperature. Concrete must have a smaller coefficient than plain mortar, since the sand and stone are unaffected, but it can hardly be less than 1/10 the above, or say 0.0001. Reinforcement will diminish this, but the shrinkage will put the steel under compression and the concrete under tension—a state of initial stress.

moment due to temperature at least should be included in this, as was done above.

Recurring to the table, we note the greatest compression to be 406 lbs. per sq. in., which is less than the usual safe allowance 500. For dead and live load only, there is no tension. With the added stresses the maximum tension is 178 lbs. per sq. in., which may not even cause a hair crack at the extrados at a_{28}. If all the stresses were 100 lbs. more, I cannot see why they are not as admissible as for the case of a reinforced beam which is expected to crack on the tension side.

It is stated in some specifications that no tension in the concrete will be allowed. This, however, refers only to the dead- and live-load stresses, and is a provision that has improved design materially, and it should be retained—with the limitation of course.

Temperature and allied stresses were not computed, the steel being supposed to provide for such. As a matter of fact, as we have seen above, the steel is stressed very little, but it furnishes a large reserve of strength to meet unforeseen emergencies — bad workmanship, yielding of the abutments, increase of loads, etc. A plain concrete arch should never be built. Shrinkage has full play

and there is no reserve of strength. A stone arch is better.

As the arch we have been examining satisfies all conditions, is economically designed, and has quite a reserve of strength, it seems unnecessary to redivide the neutral line and make a recomputation on account of the added reinforcement.

With the notation of Art. 37, Fig. 13, the arch with a clear span of 87.5 ft., rise 18 ft., measures $a=31.3$, $h=7.05$. If the original 90-ft. clear span is desired for the same h, lay off $a=\frac{90}{87.5}(31.3)=32.2$ and draw the intrados as in Art. 37. By measurement on the drawing,

$$R=76.75, \quad r=30.93.$$

To have nearly the same depth of arch ring as before, lay off the vertical depth 4.4 ft. at a_2 and a_{27} upwards from the new intrados to points. Pass a circular curve through these points and the extrados at the crown, to give the new extrados except near the ends. The radius is about 99.75 ft. The points O, O′ being 6.25 ft. above the span line and 43.54 ft. either side of the center vertical, make the sections there 7.5 ft. deep. From the upper ends of these sections, draw tangents to the new extrados, to define the extrados near the springs.

For railroad arch bridges of this type bearing heavy loads, it may suffice for a trial curve to assume the ratio of h to the rise as 7 : 18 and of a to the half-span as 32 : 90.

Shear.—The shear along any section of the arch ring is usually so small that the

concrete can take it with safety. Thus at O the shear from dead and live loads and 15° F. rise of temperature is 22,800 +5500=28,300 lbs. This acts on an area of 12×7.5×12 square inches, so that the total shear in pounds per square inch is only 27. Although this is small, yet it gives an added security to reinforce for shear, particularly to avoid possible shrinkage cracks.

CHAPTER V.

REINFORCED CONCRETE ARCH. METHOD OF SINGLE LOADS.

63. Let us consider again the arch of Fig. 11, to which will now be added a reinforcement of steel $\frac{2}{3}$ of 1% of the area of the cross-section at the crown. The span of the center line of this arch $=30$ ft., the rise 8 ft., depth at crown 2 ft., radial depth at springs 5 ft., and the earth filling extends 3 ft. above the crown. The arch ring is supposed to weigh 150 lbs. per cu. ft., and the earth 100 lbs. per cu. ft., whence the height of earth over the ring is everywhere reduced to $\frac{2}{3}$ the original height, to facilitate the computation of weights which are made out as in previous examples. The live load will be taken at 800 lbs. per sq. ft.

Assuming as usual a lamina of the arch 1 ft. thick between vertical planes perpendicular to its axis, the area of the cross-section at the crown is 2 sq. ft.;

hence the net area of the reinforcement (after deducting rivet holes) is

$$A_2 = \tfrac{2}{3} \text{ of } 1\% \text{ of } 2 = \frac{0.04}{3} \text{ sq. ft.}$$

Assume, using notation of Art. 7,

$$n = E_2 \div E_1 = 15; \quad \therefore \ 15A_2 = 0.20.$$

Calling d the radial depth of the arch at any point, and d_2 the distance between the steel bars, if these bars are placed 0.2 ft. from extrados and intrados respectively, we have

$$d_2 = d - 0.4.$$

Also, $$nI_2 = 15A_2 \frac{d_2^2}{4} = 0.05d_2^2,$$

and

$$I_1 = \tfrac{1}{12}d^3.$$

We desire to divide the neutral line so that $\dfrac{s}{I_1 + nI_2}$ or $\dfrac{s}{\frac{1}{12}d^3 + .05d_2^2}$ shall be constant, or so that $\dfrac{s}{a^3 + .6d_2^2}$ shall be constant.

To secure approximately 8 divisions of the semi-arch, take the average length of a division equal to length of neutral line from crown to spring divided by 8, or

17.68/8=2.2 ft., and take the average depth as that at the mid-point, $d=2.38$. The ratio above becomes, then, since $d_2=1.98$,

$$\frac{s}{d^3+.6d_2^2}=\frac{2.20}{21.48+.6(5.66)}=0.089.$$

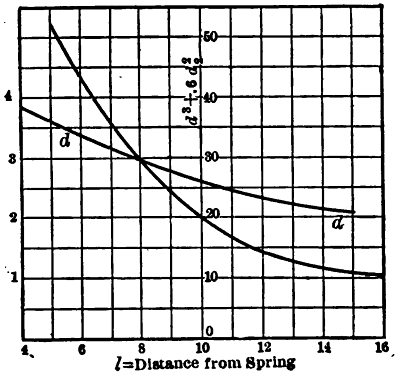

FIG. 17.

Since this is very roughly approximate, change it to 0.10, and write

$$s=0.10(d^3+0.6d_2^2),$$

where d is the radial depth at the middle of a division and $d_2=d-0.4$.

In the diagram adjoining, the length l from the spring, measured along the neutral line to a point, is laid off as an ab-

scissa and the value of d for that point (as measured from the drawing) is plotted as an ordinate; also the value of $(d^3 + 0.6d_2^3)$ is plotted as an ordinate. In practice the last expression is computed as we proceed in the following tentative method, so that only the parts of the corresponding curve actually needed are drawn. Only two or three trials are required for the first two divisions, after which the work proceeds more rapidly. The final results for a few divisions will be given to illustrate a method of keeping notes.

l = distance to middle of division.
l_e = " " end " "

Try $s_1 = 7$, $\therefore\ l = 3.5$, $d = 3.97$, $d_2 = 3.57$; $s = .1(62.57 + .6 \times 12.7) = 7.02$; $\therefore\ l_e = 7$.

Try $s_2 = 2.7$, $\therefore\ l = 7 + 1.4$, $d = 2.87$, $d_2 = 2.47$; $\therefore\ s = 2.73$; $\therefore$ take $s_2 = 2.7$; $\therefore\ l_e = 9.7$.

Try $s_3 = 1.8$, $\therefore\ l = 10.6$, $d = 2.49$, $d_2 = 2.09$; $\therefore\ s = 1.8$; $\therefore$ take $s_3 = 1.8$; $\therefore\ l_e = 11.5$, etc., etc.

We thus find

$s_1 = 7$, $s_2 = 2.7$, $s_3 = 1.8$, $s_4 = 1.4$,
$s_5 = 1.2$, $s_6 = 1.1$, $s_7 = 1$, $s_8 = 1$,

a total length of 17.20. The length of neutral line from crown to spring, measured along shorter chords than in first example, is 17.68; hence the sum of the s's must be increased 0.48, or 0.48 $\div 17.2 = .028$ per foot.

Making the increase of each s in proportion to its distance, we have the increase of s_1, $7 \times .028 = 0.196$; of s_2, $2.7 \times .028 = .076$, and so on.

The resulting values of s are:

$$s_1 = 7.20, \quad s_2 = 2.77, \quad s_3 = 1.85, \quad s_4 = 1.44,$$

$$s_5 = 1.23, \quad s_6 = 1.13, \quad s_7 = 1.03, \quad s_8 = 1.03.$$

The new "constant" for $s_1 = 7.20$ is $s_1 \div (d^3 + .6d_2^2) = 0.104$; hence

$$s = 0.104(d^3 + .6d_2^2).$$

This leads to $s_2 = 2.77$, $s_3 = 1.77$, $s_4 = 1.43$, etc.; so there is no need for making any change from the previous values.

We have seen in every example that the law of proportionate increase is justified; hence *practically* one trial determination of $s_1, s_2, \ldots$ is sufficient. The values can then be corrected by proportion as above. Considering the many approximations introduced in the theory, it seems absurd to refine too much on this division of the neutral line.

As the proposed tentative scheme need not take over two hours in the case of ten divisions, where the reinforcement is included, there seems no good reason why the reinforcement should be omitted in the computation, as is so commonly done.

The lengths $s_1, s_2, \ldots, s_8$ being laid off in turn from the springs to the crown along the neutral line, the mid-point of each length is marked a with the proper subscript. Verticals are now drawn through the a's and the crown, limited by the intrados and the reduced contour of the earth filling. The area between any two consecutive verticals multiplied by 150 gives the weight in pounds of the P corresponding to the dead load. The width between the verticals multiplied by 800 gives the live load. Any load is supposed to act midway between the verticals. The values of $P_1, P_2, \ldots, P_{16}$ are given farther on in tables, corresponding both to dead and live loads.

64. We shall first find the position of the equilibrium polygons corresponding to loads unity, acting along the same verticals as the P's. In Fig. 18 the load $P=1$ acts along the vertical through C_P a horizontal distance p from O. In the force diagram on the left, the load $P=1$ is laid off vertically, and through the top of

the load line, a horizontal $H'=1$ is laid off to fix P′, the trial pole. The trial equilibrium polygon now consists of two lines, the horizontal B_pB' and the line BB_p making an angle of 45° with the

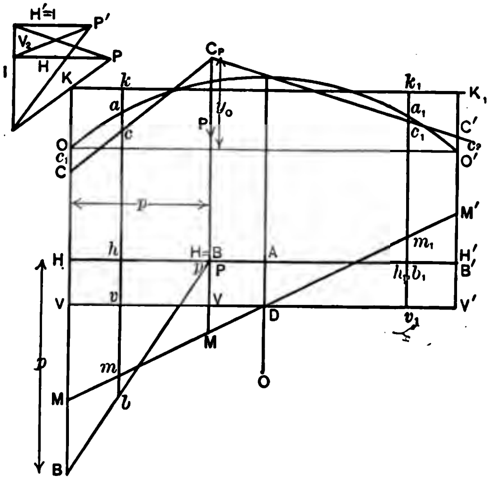

Fig. 18.

horizontal. This is most accurately effected by laying off $HB=p$. The points h of previous constructions to the right of B_p now coincide with points b, so that for such points, $bh=0$.

The constructions and formulas of preceding chapters apply here throughout.

The slight change in the demonstration is noted below.*

Observe that capital letters are used, corresponding to the points O, C_P, and O′, to avoid the possibility of inadvertently including some of these points in the summations $\Sigma(bh)$, $\Sigma(bh \,.\, z)$, $\Sigma(bh \,.\, y)$, which refer only to the points a_1 to a_{16}.

Note, too, that if the scale for the arch is such that p can be read to 0.01 ft., then bh can be scaled off within about 0.01 ft.; otherwise $bh = hB_P$ and hB_P can be measured.

The sums given in the following table are common to all the separate loadings.

---

* The only change in the demonstration of Art. 23 consists in this, that mh will be essentially negative when m lies above Ah, and the same is true of nh when n is above Ah. Thus consider two symmetrical points a and a_1 in Fig. 18, each at a distance z_1 from the vertical AD. We have

$$vh + v_1h_1 = vh + mv + v_1h_1 - m_1v_1 = mh + m_1h_1,$$

provided we regard m_1h_1 as essentially negative. As similar results obtain for any two symmetrical points, we have $\Sigma vh = \mathrm{N} \,.\, \mathrm{AD} = \Sigma(mh)$, whence $\Sigma(bh) = \Sigma(mh)$, as in Art. 23.

Next, in treating mh and m_1h_1 as forces, if mh is supposed to act down, m_1h_1, being negative, must act up, so that both must give positive moments about A. Now, $mh \,.\, z_1$ and $-m_1h_1 \,.\, z_1$ are both positive (m_1h_1 being negative), and

$$(mh - m_1h_1)z_1 = [mv + vh - (v_1h_1 - m_1v_1)]z_1 = 2mv \,.\, z_1.$$

$$\therefore\ \Sigma(mh \,.\, z) = \Sigma(mv \,.\, z).$$

The above conclusions all hold for the trial line nn_1 of Art. 23 on simply changing m to n. The subsequent deductions of that article, following eq. (i), obtain without change.

Point.	p.	y.	y^2	z.	z^2.
a_1, a_{16}	4.27	2.77	7.67	12.72	161.80
a_2, a_{15}	7.30	5.74	32.95	8.73	76.21
a_3, a_{14}	9.12	6.72	45.16	6.65	44.22
a_4, a_{13}	10.57	7.28	53.00	5.08	25.81
a_5, a_{12}	11.78	7.60	57.76	3.79	14.36
a_6, a_{11}	12.91	7.81	61.00	2.63	6.92
a_7, a_{10}	13.98	7.92	62.73	1.53	2.34
a_8, a_9	14.76	7.99	63.84	0.51	.26
		53.83 2	384.11 2		331.92 2
	$\Sigma y=$	107.66	768.22 $=\Sigma(y^2)$		663.84 $=\Sigma(z^2)$

$$e=\frac{\Sigma y}{\mathrm{N}}=\frac{107.66}{16}=6.729$$

$\Sigma(ka \,.\, y)=\Sigma(y^2)-e\Sigma y=43.67$

65. In the first table following, P = 1 is supposed to coincide with P_9. The ordinates $bh=b$, to the right, are all zero, so that the column for b_R disappears from this and subsequent tables. Similarly when P = 1 coincides in turn with P_{10}, P_{11}, etc., the ordinates bh (whether b_R or b_L) to right of P_{10}, P_{11}, etc., are all zero. The ordinates b_L to the left of P are found from the first set when P coincides with P_9, by subtracting a constant—the distance between the two positions of P = 1. Hence on determining the first set with accuracy, the other sets are determined with equal accuracy.

Point.	$P_9 = 1.$			$P_{10} = 1.$			$P_{11} = 1.$		
	b_L.	$b_L z$.	$b_L y$.	b_L.	$b_L z$.	$b_L y$.	b_L.	$b_L z$.	$b_L y$.
a_{16}	12.50	159.00	34.62	11.72	149.08	32.46	10.65	135.47	29.50
a_5	8.50	74.20	48.79	7.72	67.40	44.31	6.65	58.05	38.17
a_4	6.42	42.69	43.14	5.64	37.50	37.90	4.57	30.39	30.71
a_{13}	4.83	24.54	35.16	4.05	20.57	29.48	2.98	15.14	21.69
a_{12}	3.54	13.42	26.90	2.76	10.46	20.98	1.69	6.41	12.84
a_{11}	2.40	6.31	18.74	1.62	4.26	12.65	.55	1.44	4.29
a_{10}	1.29	1.97	10.22	.51	.78	4.03	27.09	246.90	137.20
a_9	0.26	.13	2.08	34.02	290.05	181.81			
	37.94	322.26	219.65						

$P_9 = 1.$

AD $= 2.484$. $\Sigma(mb \,.\, y) = 47.77$.

$$\frac{V_2}{1} = \frac{vm}{z} = \frac{322.26}{663.84} = 0.4854.$$

$$kc = \frac{43.67}{47.77} mb = 0.9142 mb.$$

$$H = \frac{47.77}{43.67} = 1.094.$$

$P_{10} = 1.$

AD $= 2.126$. $\Sigma(mb \,.\, y) = 47.07$.

$$\frac{V_2}{1} = \frac{vm}{z} = \frac{290.05}{663.84} = 0.437.$$

$$kc = \frac{43.67}{47.07} mb = 0.928 mb.$$

$$H = \frac{47.07}{43.67} = 1.078.$$

$P_{11} = 1.$

AD $= 1.693$. $\Sigma(mb \,.\, y) = 45.07$.

$$\frac{V_2}{1} = \frac{vm}{z} = \frac{246.9}{663.84} = 0.3719.$$

$$kc = \frac{43.67}{45.07} mb = 0.969 mb.$$

$$H = \frac{45.07}{43.67} = 1.032.$$

Point.	$P_{12}=1.$			$P_{13}=1.$			$P_{14}=1.$		
	$b_L.$	$b_Lz.$	$b_Ly.$	$b_L.$	$b_Lz.$	$b_Ly.$	$b_L.$	$b_Lz.$	$b_Ly.$
a_{16}	9.52	121.09	26.37	8.31	105.70	23.02	6.86	87.26	19.00
a_{15}	5.52	48.19	31.68	4.31	37.63	24.80	2.86	24.31	16.42
a_{14}	3.44	22.87	23.11	2.23	14.83	14.99	.78	5.01	5.24
a_{13}	1.85	9.40	13.47	.64	3.25	4.66	10.50	116.58	40.66
a_{12}	.56	2.12	4.26	15.49	161.41	67.47			
	20.89	203.67	98.89						
	AD = 1.305. $\Sigma(mb\,.\,y)=41.61.$			AD = 0.9680. $\Sigma(mb\,.\,y)=36.74.$			AD = 0.656. $\Sigma(mb\,.\,y)=29.96.$		
	$\frac{V_2}{1}=\frac{vm}{z}=\frac{203.67}{663.84}=0.3068.$			$\frac{V_2}{1}=\frac{vm}{z}=\frac{161.41}{663.84}=0.2431.$			$\frac{V_2}{1}=\frac{vm}{z}=\frac{116.58}{663.84}=0.1756.$		
	$kc=\frac{43.67}{41.61}mb=1.050mb.$			$kc=\frac{43.67}{36.74}mb=1.189mb.$			$kc=\frac{43.67}{29.96}mb=1.458mb.$		
	H = 0.9528.			H = 0.8413.			H = 0.6861.		

Point.	$P_{15}=1$.			$P_{16}=1$.		
	b_L.	b_Lz.	b_Ly.	b_L.	b_Lz.	b_Ly.
a_{16}	5.04	64.11	13.96	2.01	25.57	5.57
a_{15}	1.04	9.08	5.97	$\Sigma(b)$	$\Sigma(b.z)$	$\Sigma(b.y)$
	6.08	73.19	19.93			
	AD=0.380. $\Sigma(mb.y)=20.98$.			AD=0.1256. $\Sigma(mb.y)=7.95$.		
	$\frac{V_2}{1}=\frac{vm}{z}=\frac{73.19}{663.84}=0.1103$.			$\frac{V_2}{1}=\frac{vm}{z}=\frac{25.57}{663.84}=0.0385$.		
	$kc=2.081mb$.			$kc=5.493mb$.		
	H=0.4804.			H=0.1820.		

The general formulas used in the adjoining tables are:

$$AD=\frac{\Sigma(b)}{N}.\quad \Sigma(mb.y)=AD.\Sigma y-\Sigma(b\,.\,y).$$

$$\frac{V_2}{1}=\frac{vm}{z}=\frac{\Sigma(b\,.\,z)}{\Sigma(z^2)}.$$

$$kc=\frac{\Sigma(ka\,.\,y)}{\Sigma(mb\,.\,y)}mb.\qquad H=\frac{\Sigma(mb\,.\,y)}{\Sigma(ka\,.\,y)}.$$

66. Next in order we must compute y_0, the ordinate of C_P above OO′, as well as c_1=OC and c_2=O′C′. The method of making out the table is the same as hitherto used and will be given only for $P_9=1$. The values of *ac* for any point on the neutral line can be found similarly.

Point.	z.	VM = .485z.	MH = 2.48 ± VM.	BH.	MB = MH − BH.	KC = .914MB.	e + KC = 6.73 + KC.
O	15.00	7.27	9.75	14.76	−5.01	−4.58	+2.15 = c_1
C_p	.25	.12	2.60	0	+2.60	+2.38	+9.11 = y_0
O′	15.00	7.27	−4.79	0	−4.79	−4.38	+2.35 = c_2

The values of c_1, c_2, and y_0 thus obtained for all the single loads are entered in the following table. The values found above for H and V_2 are also entered, and the moments $M_1 = Hc_1$, $M_2 = Hc_2$ at the springs computed.

GENERAL TABLE—LOAD UNITY. $\frac{\text{Rise}}{\text{Span}} = \frac{8}{30}$.

P = 1.	c_1.	y_0.	c_2.	H.	V_2.	$M_1 = Hc_1$.	$M_2 = Hc_2$.
P_9	+ 2.15	9.11	+2.35	1.094	0.485	+2.352	+2.571
P_{10}	+ 1.82	9.12	+2.63	1.078	.437	+1.962	+2.835
P_{11}	+ 1.27	9.12	+2.96	1.032	.372	+1.311	+3.055
P_{12}	+ .56	9.13	+3.27	.953	.307	+ .534	+3.116
P_{13}	− .35	9.17	+3.54	.841	.243	− .294	+2.977
P_{14}	− 1.78	9.21	+3.85	.686	.176	−1.221	+2.641
P_{15}	− 4.23	9.29	+4.09	.480	.110	−2.030	+1.963
P_{16}	−12.88	9.70	+4.31	.182	.038	−2.344	+ .784

The equilibrium polygons corresponding to the loads P_9, P_{10}, etc., are drawn in Fig. 19.

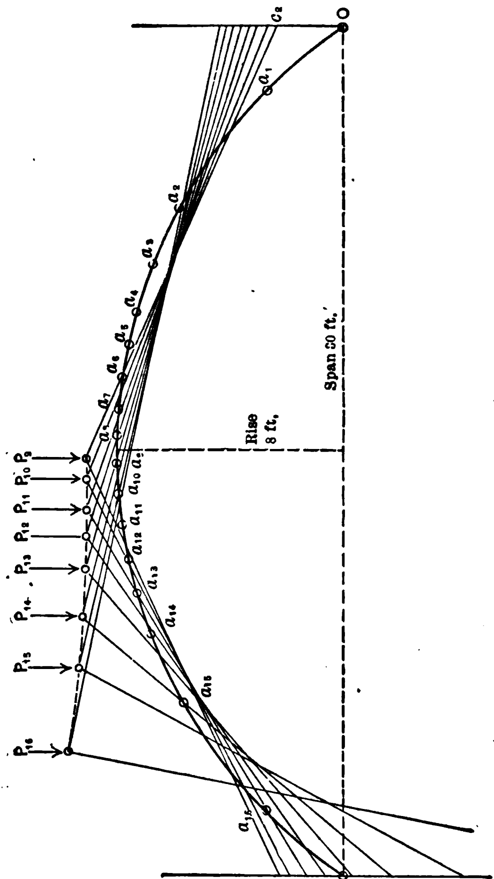

FIG. 19.

It is evident that if, in Fig. 18, a load P other than unity is considered and laid off on the load line, then to follow the construction there we must make trial H or $H' = P$. The values of bh do not change; hence AD, $\Sigma(bh \,.\, z)$, $\Sigma(bh \,.\, y)$, $\Sigma(mb \,.\, y)$ are unchanged, and the only change in the last formulas is that $\frac{V_2}{P}$ takes the place of $\frac{V_2}{1}$ and $\frac{H}{P} = \frac{\Sigma(mb \,.\, y)}{\Sigma(ka \,.\, y)}$; in words, V_2 and H are P times the values corresponding to a load unity as taken from the table. From the scheme for finding c_1, c_2, and y_0, it is plain that these quantities remain the same for any value of P.

Note.—Any load, as P_{14}, is a distributed load on a division of the arch, and hence its true equilibrium polygon over the division is a curve which falls below C_P and which is tangent to the two straight sides of the equilibrium polygon for load unity at the edges of the division or in the same verticals with the points a on either side. The moment H . ac at any point a is thus exactly given when the loads P are taken to include the part of dead or live load between verticals through consecutive a's. Other authors adopt such divisions of the arch that P acts vertically through a. The moment H . aC_P is thus considerably in error, and the equilibrium polygon resulting is not so accurately drawn.

DEAD-LOAD MOMENTS AND THRUSTS.

67. In the following table for the dead loads P, the weight in pounds of P is given in the second column, and the value of H for load unity in column 3;

VALUES OF H AND M_1+M_2 FOR DEAD LOAD.

1	2	3	4	5	6= (2)×(5).
	P(lbs.).	H coeff	H(lbs.).	M_1+M_2 coeffs.	M_1.
P_8, P_9	306	1.094	335	+4.923	+1506
P_7, P_{10}	612	1.078	660	+4.797	+2936
P_6, P_{11}	672	1.032	694	+4.366	+2934
P_5, P_{12}	741	.953	706	+3.650	+2705
P_4, P_{13}	875	.841	736	+2.683	+2348
P_3, P_{14}	1168	.686	801	+1.420	+1658
P_2, P_{15}	1803	.480	865	− .067	− 121
P_1, P_{16}	4932	.182	898	−1.560	−7694
	$V_1=11109$		5695 2		$M_1=+6272$

$H=11390$

the product of these gives the H corresponding for the successive loads. Loads, as P_8 and P_9, symmetrically placed of course have the same value of H, but the values of M_1 and M_2 are interchanged. Thus the value of M_1 for P_8 is the value of M_2 for P_9. Hence in column 5 the

values of $M_1 + M_2$ for load unity are derived from the preceding table and each multiplied by the corresponding P to give the M_1 for the two symmetrical loads.

Point O. The total dead load $M_1 = +6272$ ft.-lbs. $= M_2$ (from symmetry), and the total horizontal thrust $= H = 11{,}390$ lbs.

$$\therefore\ c_1 = c_2 = \frac{M_1}{H} = \frac{+6272}{11390} = +0.551 \text{ ft.}$$

Hence, omitting for the present the loads to the right of a_1 and to the left of a_{16}, the resultants at the springs act 0.55 ft. above O and O′.

Point a_{14}. The vertical component of the left reaction is $V_1 = 11{,}109$, the horizontal component $H = 11{,}390$, both acting at O. The couple whose moment is Hc_1 is right-handed since $Hc_1 = M_1$ is positive. Let the co-ordinates of any point a be (x, y) referred to O, x horizontal, y vertical; then if m = sum of the moments of the P's to the left of a about a, we have the general formula for the moment M_x about any point a:

$$M_x = M_1 + V_1 x - Hy - m.$$

Also, since $M_x \div H = ac$ for the point, this formula aids in fixing points of the equilibrium polygon. It applies equally where live loads are considered. For

point a_{14}, from the drawing we find $x=8.35$, $y=6.72$, and $m=1803\times1.05+4932\times4.08=22{,}015$. Substituting these values and $M_1=+6272$, $V_1=11{,}110$, $H=11{,}390$ in the formula above, we find $M_x=+484$; whence $a_{14}c_{14}=\dfrac{+484}{11{,}390}=+.04$. Since M_x is positive, the resultant from the left, acting along the side of the equilibrium polygon pertaining to a_{14}, must act *above* a_{14} to tend to turn clockwise. Thus c_{14} is 0.04 ft. above a_{14}.

Crown. The sum of the moments of $P_9, \ldots, P_{16}$ about O is found to be $\Sigma(Pp)=84{,}600$. From the table $\Sigma P=11{,}109$. Thus $\Sigma(Pp)\div\Sigma P=7.615$, or the resultant of the P's to the left of the crown acts 7.615 ft. to the right of O. The reaction at the left springing actually acts at C a distance $c_1=0.55$ ft. above O. Suppose the horizontal thrust at the crown acts h ft. above C, then on supposing the left half of the arch free and held in equilibrium by $H=11{,}390$ acting to the left at the crown, the forces P and the reaction at C, we have, on taking moments about C,

$$11{,}390h=84{,}600; \quad \therefore\ h=7.40,$$

or H acts $7.40+.55=7.95$ ft. above O, or $8-7.95=0.05$ ft. below the center of the section at the crown.

The equilibrium polygon can now be drawn as usual. The thrust at the crown and the reaction at C should intersect at a point 7.615 horizontally from O.

The dead-load moment at the crown is

$$11{,}390\times(-.05) = -570 \text{ ft.-lbs.}$$

Point c_5. From the equilibrium polygon just drawn, we find $a_5c_5 = -0.03$ ft.; hence the dead-load moment at c_5 is

$$11{,}390\times(-.03) = -342 \text{ ft.-lbs.}$$

The equilibrium polygon follows very closely the neutral line down to a_1. The straight line from a_1 to a_{16} measures 25.44 ft., the rise above a_1a_{16} to the center of the crown section is 5.23, or about $\frac{1}{5}$ the span. Hence we conclude that for segmental arches whose rise is $\frac{1}{5}$ the span or less, and with spandrel filling as assumed, the circular arc corresponds practically with the curve of equilibrium.

With regard to the thrusts normal to the sections, the most convenient way to find them is by aid of the equilibrium polygon as hitherto explained.

LIVE-LOAD MOMENTS AND THRUSTS. COMBINED RESULTS.

68. The values of the live loads P_9, etc., are given in the first column of the adjoining table. On multiplying the values of H, V_2, M_1, and M_2, given in the general table for load unity, by these values of P, we deduce the corresponding quantities for the live loads. The column for V_1, the vertical component of the left reaction, is made out by subtracting V_2 from the corresponding P.

P (lbs.).	V_1 (lbs.).	V_2 (lbs.).	H (lbs.).	M_1 ft.-lbs.	M_2 ft.-lbs.
P_9 = 408	210	198	446	+ 960	+1049
P_{10} = 816	459	357	880	+1601	+2313
P_{11} = 880	553	327	908	+1154	+2688
P_{12} = 928	643	285	884	+ 496	+2892
P_{13} = 1032	782	250	868	− 303	+3072
P_{14} = 1256	1035	221	862	−1534	+3317
P_{15} = 1664	1481	183	799	−3378	+3266
P_{16} = 3200	3078	122	582	−7501	+2509
		1943	6229		+21106

69. *Point* O. From the table, loads P_{13} to P_{16} inclusive alone give negative moments at O. The total $M_1 = -12,716$, and the total H for the same loads = 3111. The dead-load moment at O = +6272, and H = 11,390, Art. 67.

Hence total dead and live load $M_1 = -6444$; total dead and live load $H = 14,501$.

$$\therefore\ c_1 = M_1 \div H = -0.444 \text{ ft.}$$

Also, dead and live load $V_1 = 11,109 + 6376 = 17,485$.

70. *Point* O′. Maximum positive moments are caused by loads P_5 to P_{16} inclusive. The sum of the numbers in the last column gives the total M_1 for loads P_9 to P_{16}. Since total M_2 for loads P_8 to P_5 equals total M_1 for loads P_9 to P_{12}, we add to the former sum the values of M_1 for loads P_9 to P_{12} inclusive.

$$\begin{array}{lrlr}
\therefore\ \text{L. L. } M_2 = & +21,106 & \text{L. L. } H = & 6,229 \\
\text{“} \quad = & +\ 4,211 & \text{“} \quad = & 3,118 \\
\text{D. L. } M_2 = & +\ 6,272 & \text{D. L. } H = & 11,390 \\
\hline
M_2 = & +31,589 & H = & 20,737
\end{array}$$

$$\therefore\ c_2 = M_2 \div H = +1.524.$$

These are the maximum values of M, H, and c_2 for any point of the arch, as we shall see.

The live load V_2 for P_9 to P_{16} is 1943; for P_8 to P_5 it is 1865, the sum of values of V_1 for P_9 to P_{12}. Add 11,109 for dead load V_2 and we have the total vertical component of the right reaction $= 14,917$ lbs.

71. *Point* a_{14}. The moments about any point a are most accurately found from the formula of Art. 67, $M_x = M_1 + V_1 x - Hy - m$, on taking from the tables the values of M_1, V_1, and H corresponding to the loading, and computing m.

Another method of procedure will, however, be given for the remaining points. From Fig. 19 * it is seen that the sides of the equilibrium polygons corresponding to loads P_{13} to P_{16} pass *above* a_{14} the amounts 0.44, 1.56, 2.31, and 2.10 respectively, and thus give positive moments; all other loads give negative moments. The corresponding moments are found by multiplying these values of ac by the corresponding values of H taken from the last table. Thus for

P_{13}, H =	868, $ac = 0.44$;	$\therefore$ M =	382
P_{14}, H =	862, $ac = 1.56$;	$\therefore$ M =	1344
P_{15}, H =	799, $ac = 2.31$;	$\therefore$ M =	1845
P_{16}, H =	582, $ac = 2.10$;	$\therefore$ M =	1222
L. L. H =	3111	L. L. M =	4793
D. L. H =	11390	D. L. M =	484
H =	14501		+5277

$$\therefore\ a_{14}c_{14} = +5277 \div 14{,}501 = +0.364.$$

* The values of ac were scaled from a drawing on a scale of 2 ft. to the inch.

The dead-load moment + 484 was found in Art. 67. See Art. 75 for load giving maximum fiber stresses.

72. *Crown.* From Fig. 19 it is seen that live loads P_5 to P_{12} inclusive give a positive moment, the remaining loads a negative moment at the crown. We thus find positive M, as in the last case, for loads P_9 to P_{12}, and double. Similarly for H. Thus

$$\begin{array}{lrlr} \text{L. L. M} = & 2972 & \text{L. L. H} = & 6,236 \\ \text{D. L. M} = & -242 & \text{D. L. H} = & 11,390 \\ \hline \text{M} = & +2730 & \text{H} = & 17,626 \end{array}$$

$$\therefore\ ac = +2730 \div 17,626 = +0.155.$$

For loads P_{13} to P_{16} and P_1 to P_4, we find $M = -2390$, $H = 17,612$; $\therefore\ ac = -0.136$. The centers of pressure are thus well within the core points for either loading. Both loadings are very exceptional.

73. *Point a_5.* From the diagram, loads P_8 to P_{16} give negative moments. By measurement the values of a_5c_5 for the successive loads $P_8, P_9, \ldots, P_{16}$ are negative and in order:

.13, .29, .44, .61, .73, .82, .90, .93, and .96.

These are multiplied by the corresponding values of H and added. We find

$$\begin{array}{lr@{\qquad}lr} \text{L. L. M} = & -4563 & \text{L. L. H} = & 6{,}675 \\ \text{D. L. M} = & -\ \ 342 & \text{D. L. H} = & 11{,}390 \\ \hline & -4905 & & 18{,}065 \end{array}$$

$$\therefore\ a_5c_5 = -4905 \div 18{,}065 = -0.27 \text{ ft.}$$

The centers of pressure at the crown and points a_5 and a_{14} are well within the core points, so they will not be considered further.

74. *Stresses at Right Springing.*—The stresses are greatest at the right springing, since the moment and thrust both are greatest there. For loads P_5 to P_{16} we have found, for dead and live load combined, $M_2 = +31{,}589$, $H = 20{,}737$, $V_2 = 14{,}917$, and $c_2 = 1.52$. This does not include the weight of arch and filling to right of a_1 (and mainly over the spring), which must be now combined with the resultant of V_2 and H to find the true center of pressure on the section at O′. To do this, lay off on a vertical through O′ upwards $c_2 = +1.52$, say to C′. Form a triangle by laying off from C′, V_2 vertically downwards, then from the end of this line draw H horizontally to the right. The hypotenuse of this triangle

represents in magnitude and position the resultant of V_2 and H. Extend this line to intersection I with the vertical through the center of gravity of arch and filling to right of a_1, whose weight is 6078 lbs. Extend the vertical at C′ upwards 6078 to scale of V_2, and from the point thus found draw a line to the end of the line representing H. This hypotenuse represents in magnitude the resultant on the spring. On drawing through I a line parallel to the last resultant to intersection with the (radial) section at O′, we find the center of pressure there. It is 1.05 ft. radially above O′.

The component T of the final resultant is found to be 29,000 lbs.; hence the moment at O′ is $29{,}000 \times 1.05 = 30{,}450$ ft.-lbs., say 31,000.

On substituting $T = 29{,}000$, $M = 31{,}000$, $d_1 = 5$, $d_2 = 4.6$ in the formulas for stress, we derive the unit stresses in pounds per square inch in the upper fibers of the concrete and steel, 86 and 1230 compression, and in the lower fibers, 8 and 65 tension respectively. The shear over this section is only 8 lbs. per sq. in. The stresses are well within reasonable limits. The final resultant and stresses at the *left springing* are found in a similar manner. The resultant cuts the *radial* section at O; 0.25 ft. below O, $T = 27{,}600$; $\therefore M =$

6900 ft.-lbs., $t=-0.48$, $H=14{,}500$. The shear is barely 2 lbs. per sq. in.*

75. *Live Load Causing Maximum Stresses at a Section.*—In Fig. 19 conceive the arch drawn, a_1a_{16} being its axis, and consider the stresses on the upper and lower fibers of a cross-section at a_{14}. The resultant force acting on this section from any load, as P_{14}, to the *right* of a_{14} is given in position by the *left* side of the equilibrium polygon pertaining to P_{14}; the resultant for a load, as P_{15}, to the *left* of a_{14} is given in position by the *right* side of the equilibrium polygon pertaining to P_{15}. Suppose the two core points, Art. 16, note, to be marked on the section. If the left side of the equilibrium polygon corresponding to P_{12} passes *above* the lower core point, as actually happens,

* Let us compare results with those obtained in Chapter II, where only 4 divisions of the semi-arch were used and the load covered 0.8 half-span. At O, $t=-0.8$, $H=15{,}000$; $\therefore$ $M=-12{,}000$ ft.-lbs.; whereas for the case of this chapter of 8 divisions of the semi-arch, the load covering $\frac{3}{4}=0.75$ half-span, we find $t=-0.48$, $H=14{,}500$, $M=6900$.

The values of H compare favorably, since H should be slightly greater in the first case, where the live load is slightly greater. The values of t, though, are entirely too far apart. In fact we could not expect accuracy in the case of 4 divisions only. It would seem that 10 divisions of the semi-arch should be a minimum for spans up to 50 ft.

The point a_1 and its symmetrical point should not be too far from the springs, since the pressure line is the same whatever the curves between those points and the springs, provided the points O and O′ are kept upon the same level, since then y, z, and bh are unaltered.

there will be compression in the upper fiber. Hence, strictly, P_{12} must be added to the loads P_{13}, P_{14}, P_{15}, P_{16} (giving positive moments, Art. 71), to ascertain the maximum compressive stress in the upper fiber at the section. If the resultant due to P_{12} passed below the lower core point, it would cause tension in the upper fibers. In that case P_{12} must be included with all loads to its right that plainly cause tension in the upper fibers of the section at a_{14}.

Similarly we reason for the upper core point. All resultants which pass below it cause compression in the lower fibers; all resultants which pass above it cause tension in the lower fibers.

The loads causing maximum compression or tension in any fiber can thus be quickly ascertained. The diagram should be completed to include loads on the right of the crown in considering the crown and some other points. With a complete diagram, the curves touching the sides of the equilibrium polygons are called the "tangent curves." The intersection of two sides of an equilibrium polygon pertaining to the same load is a point on the "intersection locus." These curves, when drawn, enable one to find at once the reactions due to a load placed anywhere on the arch. If lines are drawn through the

two core points of a section, touching the "tangent curves," their intersections with the "intersection locus" will indicate the limits of the continuous loading for maximum stress at the edges, as explained above.

TEMPERATURE AND AXIAL STRESSES.

76. For $l=30$, $\varepsilon=.000006$, $t°=\pm 15°$ F., the change of span $= l\varepsilon t° = 0.0027$ ft. Also,

$$\Sigma y^2 - e\Sigma y = 43.67, \qquad E_1 = 2{,}000{,}000 \times 144.$$

$$\frac{I_1+nI_2}{s} = \frac{1}{12}\,\frac{d^3+.6d_2^{\,3}}{s} = \frac{1}{12\times 0.104}, \text{ Art. 63.}$$

$$\therefore\ H = \frac{E_1 l\varepsilon t°}{\Sigma y^2 - e\Sigma y} \cdot \frac{I_1+nI_2}{s} = 14{,}270 \text{ lbs.}$$

The thrust is thus much larger than for the 90-ft. span arch. This is because the present arch is so much thicker in comparison with its span.

Resolving H perpendicular and parallel to the section at O′ on a drawing, we find T=8000, shear=11,900. The moment at O′ = M = He = 14,270 × 6.73 = 96,037.

Also, $\qquad d_1 = 5, \quad d_2 = 4.6.$

The usual formula now gives for the stresses in pounds per square inch in the concrete at the right spring, for a fall of 15° F., 156 tension at extrados, 135 compression at intrados; for a rise of 15° F., 156 compression at the extrados, 135 tension at the intrados.

Axial Stress.—The normal stress per square foot at O′ is 5800, at the crown 10,350, both corresponding to dead load and to live loads P_5 to P_{16} inclusive. As a rude average take 8000; hence the change of span due to the uniform compression is

$$\frac{f}{E}l = \frac{8000 \times 30}{2,000,000 \times 144} = .00083.$$

This is $0.31 \times .0027$; hence the stresses are 0.31 times those due to a fall of 15° F. $\therefore$ at O′, tension at extrados $= .31 \times 156 = 50$; compression at intrados $= .31 \times 135 = 43$.

RESULTANT MAXIMUM STRESSES.

77. The stresses in pounds per square inch previously found in the concrete for section at O′, due to dead loads and live loads P_5 to P_{16}, forces T causing uniform compression and a change of temperature of $\pm 15°$ F., are as follows:

	D. & L. Loads.	Axial.	D. & L. & Axial.	Temp. 110° F.	Max. Comp.	Max. Tension.
O′, ext.	+86	−50	+36	±156	192	120
int.	− 8	+43	+35	±135	170	100

Compression +; Tension −.

78. *Hint as to Design.*—It is usually assumed that the arch should be so designed, that for dead load the equilibrium polygon should coincide nearly with the neutral curve down to the springs. The results above do not substantiate this view. In fact, for this arch the equilibrium polygon for dead load passes about half a foot above O and O′. When the live loads giving maximum stresses at O and O′ are added, the resultant at O is nearer O, that at O′ farther from O′, than if the dead-load polygon passed through O and O′. But in the former case, when the axial stresses are included, the stresses are +36, +35 at the edges, or nearly the same.

The axial and unknown shrinkage stresses both tend to correct the inequality of stresses +86, −8 at O′, found for dead and live loads. Again, for the dead load alone, the arch would tend to spread outwards below a_1 and a_{16}; but the earth filling can furnish a very large resistance—passive thrust—to this tendency, which is ignored in our computation.

When the earth is well compacted, it doubtless exercises very little active thrust. We conclude that, for arches of the proportions assumed in this example, the circular arc is a good one for the neutral line.

79. The final stresses above at the most stressed section are within reasonable limits, and we conclude that the arch has sufficient strength. In fact the depth at the crown could be diminished to advantage.

The amount of steel required to take all the bending moment at $O' = 31{,}000 + 96{,}000 = 127{,}000$ is found by equating this moment to

$$144 \times 36{,}000 \times \frac{A_2}{2} \times 4.6,$$

as in Art. 54, and is $A_2 = 0.0107$ sq. ft., or less than that assumed.

The reinforcement for thin highway arch bridges is sometimes as much as $\frac{3}{4}$ of 1% of the crown section, and rarely more than 1%.

POSITION OF THE LIVE LOAD CAUSING MAXIMUM MOMENTS AND STRESSES.

80. *Loading for Maximum Moments and Stresses.*—In the following table, the loading for maximum stresses (see Art. 75) as well as for maximum moments, with reference to the points indicated, is given. A uniform live load is supposed to rest on the bridge. For the crown, this extends either side of the crown, half

the distance indicated. For the other points, the live load is supposed to extend from a point vertically over O, to the right, the given distance, expressed in terms of OO′.

Point.	Length of Load for Max. Moments.	Fiber.	Length of Load for Max. Stress.
O	$0.37=\frac{3}{8}$	lower	0.41
a_{14}	$0.37=\frac{3}{8}$	upper	0.45
Crown	$0.25=\frac{1}{4}$	upper	0.39
a_5	$0.52=\frac{1}{2}+$	lower	0.56
O′	$0.63=\frac{5}{8}$	upper	0.68

For highway bridges, where the depth at crown is very much less than for this railroad bridge, the loading for maximum stresses approaches more nearly that for maximum moments. The same is true for railroad bridges for spans greater than 30 ft., to which these results apply, since the ratio of depth at crown to span decreases somewhat as the span increases. If the method of single loads is not prescribed, then for maximum stresses, at least three positions of the live load should be tried; for O and a_{14}, say 0.4 span; for a_5, 0.55 span, and for O′, 0.65 span. The crown, except when the depth there is

small, need not be investigated. The parts of the span covered by the live load can be altered slightly, to suit the divisions of the arch, without affecting materially the maximum stresses. It might be well to compute the stresses for several points in the vicinity of a_5 and a_{14}, which were arbitrarily assumed. If unequal wheel loads in place of a uniform load is specified, the labor of computation is greatly increased.*

POSITION OF LIVE LOAD GIVING MAXIMUM SHEAR.

81. In Fig. 20 let OO′ represent the neutral axis of the arch ring and X a cross-section at *a* drawn perpendicular to the axis at *a*. Also let II′ represent the intersection locus, Art. 75, and the dotted curves *t* and *t′*, the tangent locus. The shear from the left at *a* is the component of the resultant there, parallel to the section X, and will be regarded as positive when acting upwards along X, negative when acting downwards.

Draw AB tangent to the curve *t* and perpendicular to section X, to intersec-

* It is suggested in this connection that in "Specifications" for railroad arch bridges, a uniform load be required; for if actual wheel loads are specified, it adds greatly to the labor in finding maximum stresses.

tion B with II′. There is no shear along X caused by a load at B. For a load P′ to right of B, the resultant at *a* is the left reaction, which makes a less angle with the horizontal than AB. The load P′ thus causes negative shear along X. For a load P to the left of the vertical *a*C,

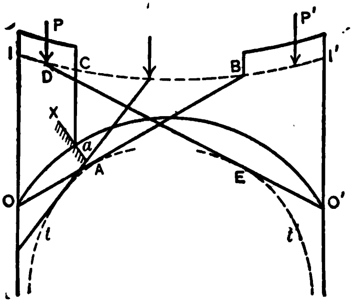

Fig. 20.

the resultant at *a* acts from D towards E and evidently causes a negative shear along X. Thus negative shear is caused by loads from I to C and from B to I′. Loads between C and B cause positive shear.

CHAPTER VI.

ARCHES WITH TWO AND THREE HINGES. BRACED ARCHES.

82. In an *arch hinged at the ends only*, suppose the neutral line to pass through the hinges A, B, Fig. 21, and that AB is horizontal. The reactions will pass through the center of the hinges if we neglect any possible friction there.

Since the span is invariable, the condition to be fulfilled by the equilibrium polygon is given by eq. (10), Art. 17:

$$\Sigma \frac{Mys}{I} = 0,$$

where M, I, and y are taken at the midpoint of the corresponding s and the summation covers the entire span. For a reinforced arch, $I = I_1 + nI_2$. Two solutions will be given. In the *first* the neutral line of the arch ring is divided into *equal* parts, each of length s, the vertical ordi-

nates y from AB to the center a of each division are measured or computed, and the value of I, at each point a, ascertained. The loads and inclined reactions form a system in equilibrium. Replace the reactions at A and B by their vertical and horizontal components V_1, H and V_2, H

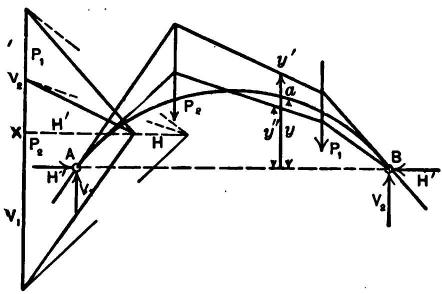

FIG. 21.

respectively. On taking moments successively about A and B of reactions and all loads acting on the arch, we find V_1 and V_2 exactly the same as for a simple beam. To compute H conceive an equilibrium polygon, due to the vertical loads and reaction components V_1 and V_2, drawn through A and B, with an assumed horizontal thrust H′. The vertical ordinate to this polygon from AB through

any point a will be called y', whence we have the known relation

$$H'y' = m,$$

where m is the moment about a of V_1 and the loads on the arch from A to a. The values of m for each point a having been computed, and H′ assumed (=10,000, say), it is seen that the value of y' for each point a can be found. It is not necessary to draw this trial equilibrium polygon. If this trial polygon were the true one, since then $M = H'(y'-y)$, the preceding condition would reduce to

$$\Sigma \frac{(y'-y)y}{I} = 0; \quad \therefore \ \Sigma\left(\frac{y'y}{I}\right) = \Sigma\left(\frac{y^2}{I}\right).$$

Both members of the last equality can be estimated.

If the equality does not obtain, alter all the ordinates of the type y' in the ratio

$$\Sigma\left(\frac{y^2}{I}\right) \div \Sigma\left(\frac{y'y}{I}\right),$$

to locate the points of the true equilibrium polygon, and alter the trial thrust H′ in the inverse ratio to ascertain the true horizontal thrust H.

The *second solution* referred to above requires the neutral axis to be divided as in Art. 19 (see also Arts. 20 and 63), thus leading to the simple condition

$$\Sigma My=0 \quad \text{or} \quad \Sigma(y'y)=\Sigma(y^2),$$

and the solution proceeds as before.*

83. If the ordinate y' has been altered as above, say to y'', then at any point a,

$$M=H(y''-y).$$

The remarks of Art. 30, relative to the proper position of the loads for the solid arch, apply equally here.

For the braced arch the loads are usually transmitted through vertical posts to fixed points on the arch, and the corresponding equilibrium polygon must of necessity be used.

The reaction at A is the resultant of V_1, previously computed, and H, just found. At any point a the resultant can at once be found from the force diagram after laying off V_1 and H as usual to fix the pole.

84. When a single force is considered, the equilibrium polygon consists of two straight lines as in Fig. 3, Art. 5; only the lines must pass through A and B, since now there are no end moments

* See a third solution in Art. 100.

there. These lines intersect on the vertical through the load. Call the ordinate of this intersection point y_0. If a *load unity* acts p feet to the right of A, the span being l feet, we have $V_1 = 1(l-p) \div l$.

Also, $$y_0 = (V_1 p) \div H' = V_1 p,$$

if H′ is assumed equal to unity. Having computed V_1 and then y_0, lay off the latter from AB upwards to fix the intersection point, from which draw lines to A and B to form the trial equilibrium polygon. The ordinates from AB to these sides through the points a, measured to scale, are the values of y'. Such ordinates to the right of the crown will be designated by y_R', those to the left by y_L'. On account of symmetry the values of y need be given only for half the arch. For the case of the neutral line being divided into, say, 20 divisions, the work can be tabulated as follows:

Points.	y.	y^2.	$\frac{y^2}{I}$.	$y'_R + y'_L$.	$\frac{(y'_R + y'_L)y}{I}$.
a_1, a_{20} a_2, a_{19} etc.					

Twice the sum of the quantities in the fourth column gives $\Sigma\left(\frac{y^2}{I}\right)$, and the sixth-column sums give $\Sigma\left(\frac{y'v}{I}\right)$ for the entire arch.

The true ordinates y'' and the true H having been found as explained above for this load unity, the values of V_1, H, and the moment $H(y''-y)$ are P times as much for a load P. Having drawn the true equilibrium polygons for loads unity at the various points, the further treatment for the solid arch is similar to that given in the previous chapter. The position of the live load corresponding to maximum stresses for the solid arch, also for maximum shears, can be found as shown in Arts. 75 and 81, noting that in the diagrams corresponding to Figs. 19 and 20, the reactions must all pass through either A or B.* The modifications for the braced arch will be indicated later.

* The diagrams of moments and shears given in Prof. C. E. Greene's "Arches" (Wiley & Sons, New York) are interesting in this connection. They refer (1) to a parabolic rib whose cross-section increases from the crown to the springing in the ratio of the secant of the inclination to the horizontal, and (2) to a circular rib of uniform section. It is seen that in either case the maximum moment at the haunches corresponds to a load covering about 0.4 of the span, and for the crown the load extends, say, 0.15 span either side of crown.

When the hinges are placed either at the upper or lower flange, the neutral line will bend sharply towards the ends to pass through the hinges A and B. The previous theory will practically apply in such cases.

TEMPERATURE AND ALLIED STRESSES FOR ARCH HINGED AT ENDS.

85. The pull or thrust due to temperature, H, acts along AB, giving moments at points a of type Hy. Suppose the neutral line of the arch to be divided into parts each of equal length s. The mid-point of any division is marked a with a proper subscript, and y is the ordinate measured from AB. Then with the notation of Arts. 56, 57, we have

$$h = l\varepsilon t^{\circ} - \frac{\Sigma(\mathrm{M}ys)}{\mathrm{EI}} = \frac{\mathrm{H}s}{\mathrm{E}}\frac{\Sigma(y^2)}{\mathrm{I}};$$

$$\therefore\ \mathrm{H} = \frac{h\mathrm{EI}}{s\Sigma(y^2)}.$$

If the span is increased a small amount h, due to a slight yielding of the abutments, the same formula applies, the reactions H at A and B now acting outwards, as in the case of a fall of tempera-

ture. The result is the same for the elastic shortening of the arch, due to the tangential forces T. For a given position of the load, these forces T can be measured on projecting the rays of the force diagram on the tangents to the neutral line at the points *a*. If the normal stress per square foot on a cross-section at any *a* due to T is f lbs., and if this is assumed to be the same throughout the length of a division *s*, the shortening of this division is $\frac{f}{\mathrm{E}}s$. If we call the horizontal projection of *s*, *z*, the change of span due to the shortening of all the divisions is

$$h = \Sigma\left(\frac{f}{\mathrm{E}}z\right),$$

which replaces the value of h in the formula above.

If f is taken as a rough average for the entire arch, we may write approximately

$$h = (fl) \div \mathrm{E},$$

where l is the span. Note here that E is the modulus in pounds per square foot when the linear unit used is the foot.

When H has been computed either for a temperature change, yielding of abutments, or elastic shortening of the arch,

the moment at any point a whose ordinate is y is given by the formula

$$M = Hy.$$

It is evidently greatest at the crown.

86. *Deflection at Crown.*—Under a symmetrical load the tangent to the neutral line at the crown is horizontal. If we regard the crown as fixed in position and the ends free, the vertical deflection of the ends will be given by the formula of Art. 18, the summation extending from the crown to one end. This is plainly the deflection of the crown when the ends are regarded as fixed. In the formula, M can represent the bending moment due to loads, temperature, etc., provided the forces acting are symmetrical with respect to the crown. The origin of co-ordinates must be taken at the left hinge, as in Fig. 8.

ARCHES WITH THREE HINGES.

87. In Fig. 22 is shown one of the best forms of arch for short spans; for in consequence of its being free to turn at three points, A, c, and B, the upper chord being cut at d and the arch proper at c, there are no stresses due to change of span, temperature, and elastic shortening

of the arch. Let us suppose that the two halves of the arch bear at c only. Suppose the right half of the arch to be loaded at the apices with the weights 1, 2, 3, and 4, due to dead and live load; the left half with the weights 5, 6, 7, due to dead load only. With an assumed pole O, draw the equilibrium polygon shown by the dotted line passing above d. Now since there can be no bending moments at A, c, and B, the actual pressure curve must pass through these points. Therefore draw a line from O parallel to Ab, the closing line of the trial curve, to intersection with the load line 17, and from this point draw a horizontal of a length equal to the old pole distance multiplied by the ratio of the ordinates $\overline{bd}$ and $\overline{ac}$, corresponding to the two curves at the center. This fixes the new pole O′, from whence the true curve passing through A, c, and B can be drawn.*

The pressure curve passes near the curved member from A to c, then keeps below the arc to B, for the loads assumed.

For a single weight, as that over apex 2, the pressure curve consists of two

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* This simple construction has been used by Professor Eddy (see "Researches in Graphical Statics") for passing a curve of pressures through three given points of a stone arch, as well as for the case above.

straight lines; one drawn from B through c to intersection with the vertical through 2, the other drawn from this last point to A. The stresses due to each weight may

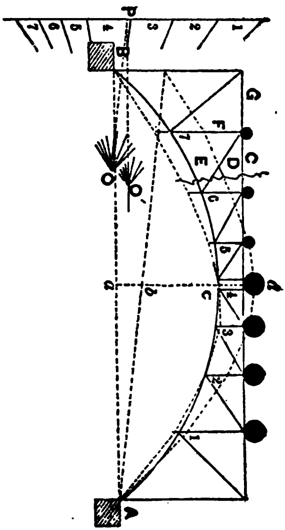

Fig. 22.

be found from its pressure curve, and tabulated, from whence the maximum stresses that any member can ever be subjected to, from the most hurtful distribution of the load, can be ascertained.

This position of the load giving a max-

imum stress in any member can likewise be found independently and then the stress in the member computed, but the limits of this little treatise forbids entering further into the subject.*

METHOD OF FINDING THE STRESSES IN THE MEMBERS OF ANY BRACED ARCH.

88. The above figure will answer, by way of illustration, for the method to be pursued in any arch. Thus, suppose for any arch shaped like Fig. 22 that we desire to know the stresses sustained by pieces C, D, and E.

Conceive a section, as shown by the wavy line, cutting C, D, and E. Suppose, now, the arch to the right of the section removed, and its action upon the left part replaced by forces acting *opposed to the resistances* in pieces C, D, and E.

The resultant of the reaction and loads *left of the section* is R = ray O′67 acting along the side of the equilibrium polygon included between the verticals at 6 and 7. Now, R acting to the right must be in

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* See on this point Merriman and Jacoby, "Roofs and Bridges," Part IV; also Balet, "Analysis of Elastic Arches." Mr. Molitor's paper on "Three-hinged Masonry arches," in *Trans. Am. Soc. C. E.*, Vol. XL, p. 56 also contains a complete discussion of the three-hinged concrete arch.

equilibrium with the *forces* applied to the cut pieces C, D, and E, which forces we may denote by the same letters as the pieces to which they correspond.

Therefore the moment of R about any point must equal the sum of the moments of C, D, and E. Thus, take 6 as a center of moments; the moments of E and D are zero, hence the moment of C about 6 equals the moment of R, from whence C can be found.

Again, take apex DG as the center of moments; we have the moment of E equal to the moment of R about DG, from whence E follows.

We see in this case that the pieces C and E are in tension and compression respectively, since the forces C and E must act from and towards the cut pieces respectively, to cause equilibrium with R.

It may be observed that when R is on the other side of the apex taken as the center of moments, that the stresses caused are of an opposite character.

The stress in D can be found by taking moments about the intersection of the pieces C and E; otherwise it can be found by taking moments about 7. The moment of R must equal the sum of the moments of C and D, etc.

Similarly, if we suppose E, F, and G cut, and the part of the arch right of the

section removed, including the weight at GD, on supplying forces E, F, and G opposed to the resistances in E, F, G, we have these forces in equilibrium with the resultant of the exteral forces to the left of the section, which is now simply the reaction at B (ray O′7P). With 7 as a center of moments, we find G, and with the apex at the left end of G as a center of moments, knowing E, we can find F; otherwise to find stress F, take the intersection of E and G as a center of moments.

The well-known Maxwell graphical method may also be employed in this case, as illustrated by Du Bois in his "Graphical Statics."

89. When the flanges of an arch are parallel, the stresses in the web members are now very easily found by decomposing R, for the section taken, into components N and T normal and parallel to the arch at the section. The normal component multiplied by the secant of its inclination to the diagonal cut gives the stress in the latter. Thus in Fig. 23 let R, acting through a, be the resultant of all the forces to the left of the section, which must therefore be in equilibrium with the forces C, D, C′ that are opposed to the resistances of the cut pieces.

Since the sum of the normal compo-

nents is equal to zero, S = N. The stress D in the diagonal is found by multiplying N by the secant of the inclination of the diagonal to the normal. The stresses in the chords are found as before.

It is seen from Fig. 23 that when N acts upwards, D is compression or tension, according as the top of the diagonal leans to the right or left of the normal; the reverse when N acts downwards.

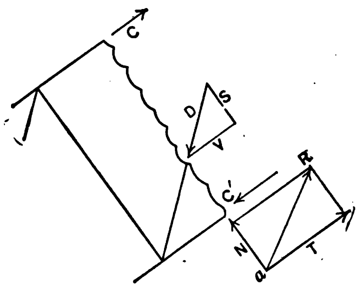

FIG. 23.

BRACED ARCHES.

90. The theory previously given in this book for hingeless arches or those with two hinges applies to the steel arch consisting of flanges connected with a solid or continuous web, provided the web is included in computing the term I. But

the theory is only approximately applicable to the braced arch with its open web, since of necessity the web cannot be considered in finding the moment of inertia I, though it is well known to have an influence in causing flexure.

If an approximate solution is admissible, then the work proceeds as usual in finding the curve of pressures, the flanges only being considered in computing the values of I at the points a.* The neutral line or axis of the arch must pass through the centers of gravity of all the cross-sections of the arch, omitting, however, the web members.

91. *Position of Live Load giving Maximum Stresses in the Members of a Braced Arch.*—Let us suppose that the braced arch has been treated for single loads, and that a diagram similar to Fig. 19 has been drawn if the arch is hingeless, or a corresponding diagram if the arch

* The braced arch can be accurately treated either by the method of deflections or that of least work. Prof. Geo. F. Swain was the first to introduce to American readers the solution by aid of the method of virtual velocities in the *Journal of the Franklin Institute* for February, March, and April, 1883.

The writer, in *Trans. Am. Soc. C. E.*, April, 1891, gave an original demonstration of the method of least work and applied it to a number of structures, including the braced arch.

As a practical aid for computing displacements, the Williot diagram has been frequently used of late. Its use saves a great deal of the laborious parts of computation.

has two hinges. The *resultant* acting at a section *a* is simply the left reaction acting along the left side of the equilibrium polygon, when the single load P considered is to the right of *a*. When P is to the left of *a*, the resultant of all forces to the left of *a* acts along the right side of the equilibrium polygon and is equal and opposed to the right reaction.

In either case the resultant of all forces to the left of *a* will simply be called the resultant at *a*.

Now conceive a section cutting only three members, as in Figs. 22 and 23, and suppose the braced arch to the right of the section removed, and that forces equal and opposed to the resisting stresses in the members cut are supplied to cause equilibrium. Then, for a center of moments, *c*, for either a chord or a web member, take the intersection of the other two members. It is plain that any load giving a resultant at the section passing above *c* will give a stress in the chord (or web member) of one character, but if the resultant passes below *c*, the stress will be of an opposite character.

This simple rule will thus indicate the kind of loading required to cause maximum stresses of either kind in a given member.

When, however, the chords are parallel at the section, they will not meet, and c for the web member does not exist. In this case the position of the loads for maximum shear is found as in Art. 81, and then the stress in the web member as in Art. 89. The previous rule applies, however, directly to the chord members and also to web members cut by a section that severs non-parallel chord members.

The braced arch, particularly the two-hinged arch, has a variety of shapes. Among these the graceful crescent form has been used both for roof trusses and bridges. For this shape the neutral line can be determined with more certainty than for more irregular shapes, especially for those where the outline is part straight and part curved.

It matters not what is the character of the curve of the neutral axis, as soon as it is even (approximately) established, the methods given in this book suffice for the ready determination of the maximum stresses in any member of the given arch.

The three-hinged arch has been used a great deal for very large roofs. The rise or fall at the crown due to temperature * is greater than for the two-hinged arch, but this is not of much importance in roof trusses; but the two-hinged roof truss is closed at the crown and, further, it can be braced against a lateral wind thrust

---

* The rise or fall at the crown due to temperature changes, for braced arches, is most easily found by aid of the Williot diagram.

more readily than a three-hinged arch; besides it is stiffer than the latter. For a bridge it is thus superior to the three-hinged arch for long spans. The hingeless arch is again stiffer than the two-hinged arch. If the abutments are practically immovable, it is the best form to use, though the workmanship required is of a very high order to ensure the truss fitting the span properly.

THREE-HINGED ARCH ROOF TRUSS, WIND LOAD INCLUDED.

92. In addition to the dead and snow loads, a wind load will be considered in treating a three-hinged arch roof truss.

If we suppose the wind to blow horizontally with a force P against a square foot of vertical plane, the experiments of Hutton go to show that the normal pressure on a square foot of roof surface inclined at an angle i to the horizontal equals

$$N = P \sin i^{1.84 \cos i - 1}.$$

More extended experiments are needed to verify this formula, but assuming it to be true, we have, taking $P = 40$ lbs. per sq. ft. as the greatest intensity of the wind in a horizontal direction likely to occur, the following values of N in pounds for different inclinations of the roof surface, taken from Greene's "Roof Trusses":

i.	N.	*i*.	N.	*i*.	N.
5°	5.2	25°	22.5	45°	36.1
10	9.6	30	26.5	50	38.1
15	14.0	35	30.1	55	39.6
20	18.3	40	33.4	60	40.0

For steeper slopes N is 40 lbs.

93. Let ACB, Fig. 24, represent the neutral line of a pointed roof truss, the arc AC being described with a center *d*, the arc BC with a center *d′*. In this figure each half-arc is divided into 16 equal parts, and ordinates drawn through the center of each division.

Assume the slope of roof *i* on each division to be that of a tangent to the center of each division, then the force of wind on the 15th division, inclined 80° to the horizon, is 40 lbs., and it acts in the direction of the radius $\overline{15d}$. Hence lay off $\overline{dm}$ on the radius produced equal to 40 lbs. multiplied by area of division; if $\overline{dn}$ represents the weight of one division of roof, $\overline{nm}$ is the resultant oblique force acting at the center of division 15, through which a line is drawn parallel to $\overline{nm}$.*

* The weight of each division, including purlins, sheeting, snow, etc., does not generally act through the center of the division of the neutral line; hence it is

94. Proceeding in this manner for each division, values of N being interpolated from the table when necessary, we next lay off on a force diagram on the left, beginning at *e*, the forces 1, 2, . . . , acting at the center of divisions 1, 2, ... in order and parallel to their directions, Similarly, on the vertical through *e′*, lay off the equal weights on the divisions from C to B in order.

(The forces are here laid off to a smaller scale than that to which $\overline{nm}$ was drawn, for convenience.)

The lines $\overline{ef}$ and $\overline{e'f'}$ represent, therefore, the intensity and directions of the resultants of the forces just found from C to A and from C to B respectively.

The positions of these resultants P_1 and P_2 are found as follows: The line *eCe′* was drawn in the first instance passing through the crown, at about the inclination it was thought the pressure curve would have there. Hence, assuming O and O′, equally distant from *e* and *e′*, as poles, draw the pressure curves for the left and right halves of the arch respectively, starting at C with the assumed inclination of the thrust there. The

most accurate to combine N with the vertical load acting through its center of gravity, so that the oblique line just drawn will be moved slightly to the left. The subsequent constructions are the same in either case.

curve on the right is drawn as usual and needs no explanation. It lies very near the neutral line at first and then passes above it. On prolonging the last side to intersection E′ with Ce', we find the position of the resultant P_2 of the forces on the right half of the arch.

For the left side we extend Ce to intersection with force acting at 1, then draw a line ∥ ray 12 to force at 2, then a line ∥ ray 23 to force at 3, and so on. On prolonging the last side to E, we have $EP_1 \parallel ef$, the position of the resultant $P_1 = \overline{ef}$ of the oblique forces acting on the left half of the arch.

95. It is evident, so long as the loading remains the same, that the positions and magnitudes of P_1 and P_2 remain the same for any pressure curve. It is now very easy to find the reaction at A in order that a pressure curve may pass through the three points A, C, and B. Thus call the vertical and horizontal components of the reaction at A, V and H respectively. If B is at the same level as A, on taking moments about B, we have

$$V \cdot \overline{AB} = P_1 p_1 + P_2 p_2,$$

calling p_1 and p_2 the length of perpendiculars from B upon P_1 and P_2 respectively.

To find H take moments about the crown of the forces to the left of it. Thus calling the length of the perpendiculars from C to P_1 and AB, c_1 and h respectively, we have

$$Hh = V\frac{AB}{2} - P_1c,$$

from whence H is found.

Laying off now $\overline{fg} = V$ and $\overline{gD} = H$, we have D as the new pole on the left. On drawing $e'D'$ parallel and equal to eD, we have D′ for the position of the new pole for the forces acting on the right half of the arch.

The direction of the pressure at C is the line FcF' parallel to eD or $e'D'$.

Starting at C we draw the pressure curve as before. It is shown by the dotted line passing through A, C, and B.

It is well to test the computed values of V and H before drawing the pressure curve, by drawing through A and B lines parallel to fD and $f'D'$. If these lines intersect FF′ at the same points F and F′ with P_1 and P_2, the poles D and D′ have been correctly found.

The above pressure curve is the true one for the roof truss hinged at A, C, *and* B.

TWO-HINGED ARCH ROOF TRUSS, WIND LOAD INCLUDED.

96. Regard Fig. 24 now as an arch hinged only at A and B. The vertical components of the reactions remain the same as for the three-hinged arch, as we see from the first formula of the preceding article, but H must be altered to satisfy the conditions of continuity at the crown. To take the most general case, suppose I to vary, but that s is constant as in the figure. Let a be the mid-point of any division s and suppose I and y refer to this point and that M is the moment about a of all external forces left of this point; then the condition that the span should be invariable, Art. 17, reduces to

$$\Sigma\left(\frac{My}{I}\right)=0 \quad . \quad . \quad . \quad . \quad (1)$$

the summation covering the entire arch.

Suppose now for a trial polygon passing through A and B that M′, the moment about a of all external forces left of a, has been computed for each point a and that the summation for the entire span gives

$$\Sigma\frac{M'y}{I}=A \quad . \quad . \quad . \quad . \quad (2)$$

Any moment will be regarded as positive when it corresponds to a right-handed couple, or when the resultant of the forces left of a, acting along the proper side of the equilibrium polygon, acts above a. Therefore if A is positive, it shows that the polygon is too high and that H′, the horizontal thrust, must be increased. The reverse obtains when A is negative.

If, now, two opposed horizontal thrusts H″ are added at A and B, the moment M″ of H″ at A about any a is

$$M'' = H''y.$$

Let H″ be determined by the equation

$$H''\Sigma\frac{y^2}{I} = \Sigma\frac{M''y}{I} = -A; \quad . \quad . \quad (3)$$

then when A is +, H″ is −; when A is −, H″ is +. Therefore when A is +, M″ is −, and H″ acts inwards; when A is −, M″ is +, and H″ acts outwards, to agree with the convention above as to the signs of a moment.

Let $\qquad M = M' + M'';$

then on adding (2) and (3), we derive (1), and the true thrust is

$$H = H' - H'',$$

since we have just seen that when H'' is positive, it must act outwards; when negative, inwards.

We have then to compute A or the left member of (2), also $\Sigma(y^2/I)$, and determine H'' from (3); then the true moment at any point a is

$$M = M' + M'' = M' + H''y,$$

since (1) is satisfied by such values of M as is shown above.

97. Recurring to the pressure curve ACB, having the poles D and D', we have the *bending moment* about any point on the neutral axis between 4 and 5, say, equal to ray D45, measured to the scale of force multiplied by the perpendicular distance from the point to the resultant acting along the side 45 of the *pressure line.*

This moment is also equal to the horizontal component H' of ray D45, multiplied by the vertical distance v from the point to the resultant. This is evident if we decompose the resultant at a point of its line of action vertically over the point taken in the neutral axis into vertical and horizontal components. The latter alone causes a moment about the point equal to $H'v$, as stated.

The loads in Fig. 24 were taken as acting at the points a or 1, 2, . . . , for simplicity. They should preferably be supposed to act between these points.

As the oblique forces are inclined inwards, it is seen that if we measure the vertical distances v from 1, 2, . . . to the pressure line as usual, that when the pressure line is above the neutral line, H′ must be taken as the horizontal component of the resultant acting just to the *right* of the force acting at the point taken; otherwise, when the pressure line is below the neutral line, H′ corresponds to the resultant acting just to the left of the force.

Thus at 4 use the ray D34; at 13 the ray (D13, 14), in evaluating H′, v being measured vertically from 4 and 13 to the pressure line. As usual v is + when above the neutral line, − below it, in finding

$$\Sigma\left(\frac{M'y}{I}\right).$$

The moments M to the left of C are of the type H′v, H′ and v both being variable; to the right of C, M = H′v, v alone being variable, H′ representing the constant pole distance from D′ to force line $e'f'$. The ordinates y are, of course, measured from AB to the neutral line of the rib.

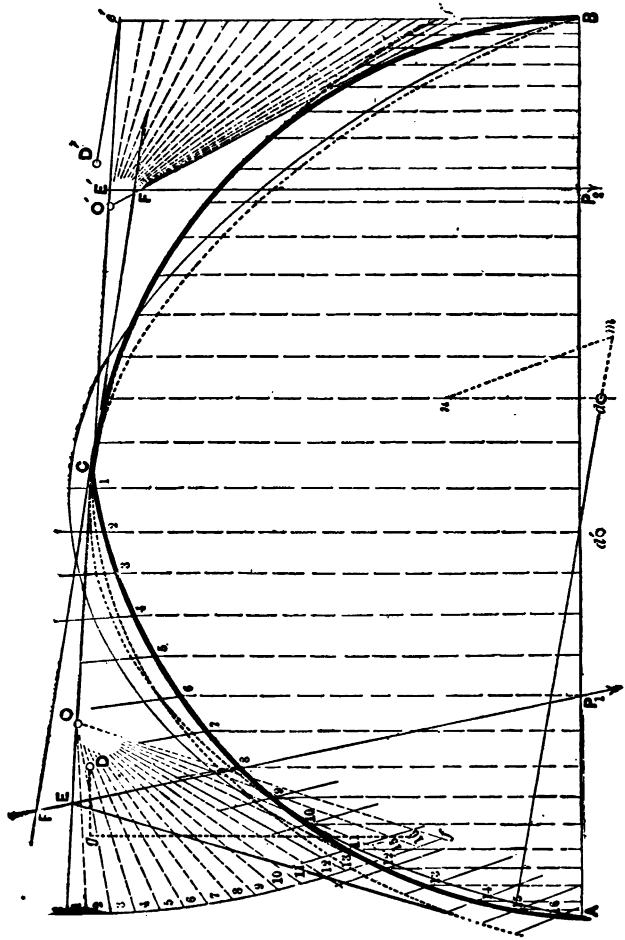

Fig. 24.

98. As a numerical illustration, I was taken as constant for the arch shown, so that it drops out of (1) and the following equations. The "trial pressure curve" passed through A, B, and about 1/20 inch above C, and to a certain scale it was found that

$$\Sigma M'y = \Sigma H'vy = -1000,$$

the summation including the entire arch. Also, $\Sigma y^2 = 1500$; $\therefore$ by (3), $1500H'' = +1000$. $\therefore H'' = +\frac{2}{3}$. Hence H″ acts outwards at both A and B. Therefore to the scale of force, H″ was laid off from the left trial pole to the left, to fix the true pole there. The new position of the right pole is then found as before, and the true pressure curve drawn. It is shown by the full line in the figure passing through A, B, and a point about 1/10 inch above C.

99. If the neutral line is divided into such lengths s_1, s_2, ... that s/I is the same for each division, then I is dropped from equations (1), (2), (3) of Art. 96, and the solution is simplified as just illustrated.

100. For vertical loads only, the equations of Art. 96 equally apply and lead to the results of Art. 82. Thus, adopting the notation of the latter article, and

since H′ is now constant, eq. (2) of Art. 96 reduces to

$$H'\Sigma\frac{(y'-y)y}{I}=A.$$

On substituting this value of A in (3), solving for H″, and substituting the value of H″ in H = H′ − H″, we derive

$$H=H'\Sigma\left(\frac{y'y}{I}\right)\div\Sigma\left(\frac{y^2}{I}\right),$$

which is exactly the value of H, the true horizontal thrust, obtained otherwise in Art. 82.

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